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Binary PSO Based Dynamic Multi-objective model for DG Planning under Uncertainty

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Abstract

This paper proposes a dynamic multi-objective model for integration of distributed generations in distribution networks. The proposed model optimizes three objectives, namely technical constraint dissatisfaction, costs and environmental emissions and simultaneously determines the optimal location, size and timing of investment for both DG units and network components. The uncertainties of electric load, electricity price and wind power generations are taken into account using scenario modeling. A scenario reduction technique is used to reduce the computational burden of the model. The Pareto optimal solutions of the problem are found using a binary PSO algorithm and finally a fuzzy satisfying method is applied to select the optimal solution considering the desires of the planner. The effectiveness of the proposed model is demonstrated by applying it to a realistic 201-node distribution network.

Index Terms

Distributed generation, PSO, Dynamic planning, scenario reduction, Pareto optimal front.

NOMENCLATURE

Constants

<table>
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<th>Symbol</th>
<th>Description</th>
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<tr>
<td>( C_\ell )</td>
<td>Cost of reinforcement of feeder ( \ell )</td>
</tr>
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<td>( C_{tr} )</td>
<td>Cost of investment in transformer</td>
</tr>
<tr>
<td>( d )</td>
<td>Discount rate</td>
</tr>
<tr>
<td>( E_{grid/dg} )</td>
<td>Emission factor of the grid/dg</td>
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IC\textsubscript{dg}, OC\textsubscript{dg} \quad \text{Investment and operating cost of a } dg

T \quad \text{Planning horizon}

\alpha \quad \text{Rate of demand growth}

\textbf{Variables}

\begin{align*}
P_{D,i,t,dl,s} & \quad \text{Active power demand in bus } i, \text{ in year } t, \text{ demand level } dl \text{ and state } s \\
P_{\text{grid}, t,dl,s} & \quad \text{Active power purchased from grid in year } t, \text{ demand level } dl \text{ and state } s \\
P_{\text{dg}, i,t,dl,s} & \quad \text{Active power injected by a } dg \text{ in bus } i, \text{ in year } t, \text{ demand level } dl \text{ and state } s \\
S_{\text{grid}, t,dl,s} & \quad \text{Apparent power imported from grid in year } t, \text{ demand level } dl \text{ and state } s \\
S_{\text{dg}, i,t,dl,s} & \quad \text{Apparent power of } dg \text{ installed in bus } i, \text{ in year } t, \text{ demand level } dl \text{ and state } s \\
I_{\ell,t,dl,s} & \quad \text{Current magnitude of } \ell^{th} \text{ feeder in year } t, \text{ demand level } dl \text{ and state } s \\
v_{\text{cut}} & \quad \text{Cut-in speed of wind turbine} \\
v_{\text{cut}} & \quad \text{Cut-out speed of wind turbine} \\
\mu_{V,i,t,dl,s} & \quad \text{Degree of voltage constraint satisfaction for bus } i, \text{ in year } t, \text{ demand level } dl \text{ and state } s \\
\mu_{V,i,t} & \quad \text{Degree of voltage constraint satisfaction for bus } i, \text{ in year } t \\
\mu_{V,t} & \quad \text{Degree of voltage constraint satisfaction for whole system in year } t \\
\mu_{I,\ell,t,dl,s} & \quad \text{Degree of thermal constraint satisfaction for feeder } \ell \text{ in year } t, \text{ demand level } dl \text{ and state } s \\
\mu_{I,\ell,t} & \quad \text{Degree of thermal constraint satisfaction for feeder } \ell \text{ in year } t \\
\mu_{I,t} & \quad \text{Degree of thermal constraint satisfaction for whole system in year } t \\
\mu_{S}\text{grid}_{t,dl,s} & \quad \text{Degree of overall thermal constraint satisfaction for substation in year } t \text{ and state } s \\
\mu_{k}(X_{n}) & \quad \text{Degree of minimization satisfaction of } k^{th} \text{ objective function by solution } X_{n} \\
S_{D,i,t,dl,s} & \quad \text{Demand in bus } i, \text{ year } t, \text{ demand level } dl \text{ and state } s \\
EP_{s} & \quad \text{Electricity price in state } s \\
FN_{n} & \quad \text{Front number to which } n^{th} \text{ solution belongs} \\
GD_{n} & \quad \text{Global Diversity of } n^{th} \text{solution} \\
\gamma_{\ell,t} & \quad \text{Investment decision in feeder } \ell, \text{ in the year } t \\
\xi_{i,t} & \quad \text{Investment decision for DG technology } dg \text{ in bus } i, \text{ in the year } t \\
\psi_{t} & \quad \text{Investment decision in transformer, in the year } t
\end{align*}
\( d_\ell \) Length of feeder \( \ell \) in km

\( LD^k_n \) Local diversity of \( n^{th} \) solution in \( k^{th} \) objective function

\( MD_k \) Maximum difference between the values of \( k^{th} \) objective function, regarding all solutions

\( \overline{S}_{lim}^{dg} \) Maximum operating limit of a \( dg \)

\( P_{net}^{i,t,dl,s} \) Net active power injected to bus \( i \), in year \( t \), demand level \( dl \) and state \( s \)

\( Q_{net}^{i,t,dl,s} \) Net reactive power injected to bus \( i \), in year \( t \), demand level \( dl \) and state \( s \)

\( N_b \) Number of buses in the network

\( N_p \) Number of population

\( N_\ell \) Number of feeders in the network

\( N_o \) Number of objective functions

\( N_J \) Number of combined states

\( N_s \) Number of reduced states

\( \rho \) Peak price of energy purchased from the grid

\( S_{i,peak}^{D} \) Peak demand in bus \( i \), in state \( s \)

\( \lambda_{dl,s} \) Percent of peak electricity price in state \( s \) and demand level \( dl \)

\( D_{dl,s} \) Percent of peak electricity demand in state \( s \) and demand level \( dl \)

\( wp_s \) Percent of rated capacity of installed wind turbine in state \( s \)

\( P_{w,r}^{i,r} \) Rated power of installed wind turbine

\( Q_{dg}^{i,t,dl,s} \) Reactive power injected by a \( dg \) in bus \( i \), year \( t \), demand level \( dl \) and state \( s \)

\( Q_{D}^{i,t,dl,s} \) Reactive power demand in bus \( i \), year \( t \), demand level \( dl \) and state \( s \)

\( SF_n \) Pseudo fitness of \( n^{th} \) particle

\( \text{prob}_{s}^{l} \) Probability of load state \( s \)

\( \text{prob}_{s}^{\lambda} \) Probability of electricity price state \( s \)

\( \text{prob}_{s}^{w} \) Probability of wind speed state \( s \)

\( \text{prob}_{s}^{c} \) Probability of combined state \( s \)

\( v_{\text{rated}} \) Rated speed of wind turbine

\( c \) Scale factor of Rayleigh PDF of wind speed

\( TD_t \) Technical dissatisfaction in year \( t \)

\( \text{GC} \) Total cost paid to grid

\( \text{DGIC} \) Total Investment cost of DG units
I. INTRODUCTION

A. Motivation and problem description

Distributed generation (DG) is an electric power source connected directly to the distribution network [1]. There are five major factors that motivate increasing the share of DG units in electricity generation: electricity market liberalization, development in DG technology, constraints in the construction of new transmission lines, reliability enhancement [2] and concerns about the environment [3]. DG may also offer distribution network operators (DNOs) more diverse, flexible, and secure options for managing their electricity systems to benefit customers [4]. A powerful tool for planners is needed to model the uncertainties of intermittent power generations of wind turbines and also electric load and price values. It should not only reduce the computational burden but also maintain the accuracy of computation. The motivation of this study is to provide such a tool.

B. Literature review

In recent years, many approaches have been proposed to solve the DG planning problem. The literature suggests a wide range of objectives, such as voltage stability improvement [5], risk aversion in load procurement [6], active loss reduction [7], [8], reactive loss reduction [9], reliability improvement, reducing the cost of energy required for serving the customers, increasing the incentives received by distribution network owners for using DGs, reducing the
cost of energy not supplied, injecting power into the grid at peak load and emission reduction [10]. These studies have considered a variety of technical issues including voltage profile [11], [12], thermal limits of conductors [13], substation capacity [14], three phase and single phase to ground short circuit [13], [15], and load modeling [9]. The reported models for DG planning can be divided into two major categories: static and dynamic models. In static models, investment decisions are implemented in the first year of the planning horizon [5], [9], [16]. In this category, the models are single or multi-objectives. The single-objective models are either originally single-objective [5], or multi-objective which are converted into a single-objective (using a benefit to cost ratio index or an additive utility function [9]); multi-objective models of this category are solved using Pareto optimality concept [17]. The dynamic models of the literature are those which determine the optimal investment decisions to be made at each year (or time segment) during the planning horizon [18]. These models consider the time value of money and are more efficient than static models. The planning models are summarized in Table I.

C. Contributions

In this paper, a dynamic multi-objective DG-planning problem is formulated and a two-stage algorithm is proposed to solve the problem. In the first stage, the set of Pareto optimal solutions is found using a novel binary PSO method, and in the second stage, the best solution is chosen using fuzzy satisfying techniques. The model aims to cover all three aspects of DG planning problem, i.e., siting, sizing and timing of investment simultaneously, in order to increase the technical, economical and environmental benefits accrued by a DNO. The contributions of this paper are three fold:

1) Multi-objective dynamic modeling of DG planning problem for simultaneous determination of timing, sizing and siting, when costs, emissions and technical attributes of the proposed plans, and a variety of DG technologies are considered.

2) The uncertainties of input variables are taken into account.

3) Proposing a BPSO method for solving the aforementioned problem.

D. Paper organization

This paper is set out as follows: section II presents problem formulation, section III sets out the principles of multi-objective optimization and the proposed solution algorithm for solving
II. Problem Formulation

The multi-objective DG planning problem is formulated in this section. The decision variable is the number of DG units from a specific technology, to be installed in bus \( i \) in year \( t \), i.e., \( \xi_{i,t}^{dg} \). Investment in feeders, i.e. \( \gamma_{t}^{f} \), or transformer, i.e. \( \psi_{t}^{tr} \).

A. Assumptions

The following assumptions are employed in problem formulation:

- Connection of a DG unit to a bus is modeled as a negative PQ load with a fix power factor.
- DNO is authorized to invest in DG units.

B. Uncertainty modeling

The electricity price and electric load are both uncertain in deregulated environment but these parameters are specifically tied together. An increase/decrease in electric load will tend to increase/decrease in electricity price. Without loss of generality, the correlation between wind speed and load-price pattern are assumed to be independent [19]. If any correlation exists between load-price and wind pattern this can be easily considered in the proposed algorithm. The price and load duration curves are divided into \( N_{dl} \) levels in each year as shown in Fig. 1. The vertical axis in Fig. 1, shows the demand/price level factors (the ratio of load/price to the peak value of load/price in this level). The duration of each level is described by \( \tau_{dl} \). It is assumed that the demand/price level factors (\( \lambda_{dl}, D_{dl} \)) are normally distributed around their specified expected values as shown in Fig. 1. Each normal distribution is divided into 7 states and the probability of each state is specified in Fig. 1. Although the expected price and demand values are dependent but ,in each demand level, the variation of price and electric load around its expected value can be assumed to be independent. The electricity price, electric load and wind generation are modeled as follows:
1) **Electricity price:** The price of energy purchased from the grid is competitively determined in a liberalized market environment. Assuming a peak electricity price of \( \rho \), the electricity price in demand level \( dl \), and state \( s \) can be calculated as:

\[
EP_{dl,s}^\lambda = \rho \cdot \lambda_{dl,s}
\]  

(1)

2) **Electric load:** Assuming a peak load of \( S_{i,peak}^D \) and a demand growth rate of \( \alpha \), the demand in bus \( i \), in year \( t \), demand level \( dl \) and state \( s \) can be calculated as:

\[
S_{i,t,dl,s}^D = S_{i,peak}^D \cdot D_{dl,s} \cdot (1 + \alpha)^t
\]  

(2)

3) **Wind speed and wind turbine power generation:** The generation schedule of a wind turbine highly depends on the wind speed in the site. There are various methods to model wind behavior. In this paper, the variation of wind speed, i.e. \( v \), is modeled using a Rayleigh PDF and its characteristic function which relates the wind speed and the output of a wind turbine.

\[
PDF(v) = \left( \frac{2v}{c^2} \right) exp\left[-\left(\frac{v}{c}\right)^2\right]
\]  

(3)

where \( c \) is the scale factor of the Rayleigh PDF of wind speed in the zone under study.

The generated power of the wind turbine is determined using its characteristics as follows:

\[
P_{i,t}^{w}(v) = \sum_{l=1}^{t} \xi_{i,t}^{dq} \cdot \begin{cases} 
0 & \text{if } v \leq v_{in}^c \text{ or } v \geq v_{out}^c \\
\frac{v-v_{in}^c}{v_{rated}-v_{in}^c} \cdot P_{i,r}^{w} & \text{if } v_{in}^c \leq v \leq v_{rated} \\
P_{i,r}^{w} & \text{else}
\end{cases}
\]  

(4)

Where, \( P_{i,r}^{w} \) is the rated power of wind turbine installed in bus \( i \), \( P_{i,t}^{w} \) is the generated power of wind turbine in bus \( i \), \( v_{out}^c \) is the cut out speed, \( v_{in}^c \) is the cut in speed and \( v_{rated} \) is the rated speed of the wind turbine. The speed-power curve of a typical wind turbine is depicted in Fig. 2. Using the technique described in [19], the PDF of wind speed is divided into several states. In each state, the probability of falling into this state is calculated as follows:

\[
prob_s^{w} = \int_{V_{1,s}}^{V_{2,s}} (2v) \left( \frac{2v}{c^2} \right) exp\left[-\left(\frac{v}{c}\right)^2\right] dv
\]  

(5)

\[
v_s = \frac{V_{2,s} + V_{1,s}}{2}
\]

The generated power of wind turbine is calculated using the \( v_s \), as obtained in (5), and (4).
4) Combined states model: As it is already mentioned, the states of each demand level are independent (the correlation between load and price is already considered in their mean value of $D_{dl}$ and $\lambda_{dl}$). In each demand level, the states are combined to construct the whole set of states as follows:

$$C(s) = \text{load}(s) \cdot \text{price}(s) \cdot \text{wind}(s)$$

(6)

$$\text{Prob}_s^c = \text{prob}_s^l \cdot \text{prob}_s^\lambda \cdot \text{prob}_s^w$$

(7)

where $\text{Prob}_s^c$ is the probability of each combined state.

C. Constraints

1) Power flow constraints: The flow equations that shall be satisfied for each configuration and states are:

$$P_{\text{net}}^{\text{net}}_{i,t,dl,s} = -P_{i,t,dl,s}^{D} + P_{i,t,dl,s}^{dg}$$

(8)

$$Q_{\text{net}}^{\text{net}}_{i,t,dl,s} = -Q_{i,t,dl,s}^{D} + Q_{i,t,dl,s}^{dg}$$

$$P_{i,t,dl,s}^{net} = V_{i,t,dl,s} \sum Y_{ij}^{t} V_{j,t,dl,s} \cos(\delta_{i,t,dl,s} - \delta_{j,t,dl,s} - \theta_{ij})$$

$$Q_{i,t,dl,s}^{net} = V_{i,t,dl,s} \sum Y_{ij}^{t} V_{j,t,dl,s} \sin(\delta_{i,t,dl,s} - \delta_{j,t,dl,s} - \theta_{ij})$$

2) Operating limits of DG units: Each DG should be operated considering its capacity limits, i.e.:

$$S_{i,t,dl,s}^{dg} \leq \sum_{l=1}^{t} \xi^{dg}_{i,t} \cdot S_{lim}^{dg}$$

(9)

3) Fuzzy technical satisfaction: The satisfaction of soft constraints can be modeled by fuzzy sets. The idea of fuzzifying the technical constraints was used by [20]. In the present work, this idea is extended to model the problem with different states for a dynamic planning problem. Fuzzy modeling is used to quantify the value of satisfaction of technical constraints of voltages and thermal limits of feeders and substation, as follow:

a) Voltage profile: The voltage magnitude of each bus should be kept between the safe operation limits. However, the DNO may ignore violation of these limits to some degree, in hope of achieving a better solution regarding other necessities [20]. The membership function of the voltage constraint satisfaction is represented by a trapezoidal fuzzy number [17].
Observe that a voltage magnitude between the up and low safe operation limits, i.e., $V_{\text{min safe}}$, $V_{\text{max safe}}$, has a satisfactory value of 1. As the voltage exceeds these limits, the value of satisfaction decreases until it becomes zero after the critical voltage values, i.e., $V_{\text{min crit}}$, $V_{\text{max crit}}$. This function can be mathematically represented as:

$$
\mu_{V_{i,t,dl,s}} = \begin{cases} 
\frac{V_{\text{min safe}} - V_{\text{min crit}}}{V_{\text{safe}} - V_{\text{crit}}} & V_{\text{crit}} \leq V_{i,t,dl,s} \leq V_{\text{min safe}} \\
1 & V_{\text{min safe}} \leq V_{i,t,dl,s} \leq V_{\text{max safe}} \\
\frac{V_{\text{max safe}} - V_{\text{max crit}}}{V_{\text{safe}} - V_{\text{crit}}} & V_{\text{max crit}} \leq V_{i,t,dl,s} \leq V_{\text{max safe}} \\
0 & \text{else} 
\end{cases}
$$

The values obtained from (10) show the condition of voltage constraint satisfaction for bus $i$ in state $s$ in year $t$. Since there are more than one state in a real system, the planner will have different satisfaction levels of voltage constraint for a given bus. To obtain an index which shows the condition of a given bus $i$ in year $t$, it is proposed in this work to calculate the weighted average of satisfaction of voltage over the states, as follows:

$$
\mu_{i,t} = \frac{1}{8760} \sum_{dl=1}^{N_{dl}} \sum_{s=1}^{N_{s}} \text{prob}_s \cdot \tau_{dl} \cdot \mu_{V_{i,t,dl,s}}
$$

In (11), if the voltage of bus $i$ does not fully satisfy the constraints in state $s$ but the probability of this dissatisfaction is short, the satisfaction of this bus is not very degraded in the whole year $t$. The average value of $\mu_{V_{i,t}}$ over all buses of the network, can provide information about the overall voltage condition in year $t$ as follows:

$$
\mu_{V} = \frac{\sum_{i=1}^{N_b} \mu_{i,t}}{N_b}
$$

b) Thermal limit of feeders and Substation: To maintain the security of the feeders and the substation, the flow of current/energy passing through them should be kept below the feeders/substation capacity limit. This is incorporated here, in the form of a fuzzy membership function [17]. A strictly monotonically decreasing and continuous function is considered for this
limit, as follows:

\[
\mu_{I_{\ell,t},dl,s} = \begin{cases} 
1 & I_{\ell,t,dl,s} \leq T_{\text{safe},t} \\
\frac{I_{\ell,t,dl,s} - T_{\text{crit},t}}{T_{\text{safe},t} - T_{\text{crit},t}} & T_{\text{safe},t} \leq I_{\ell,t,dl,s} \leq T_{\text{crit},t} \\
0 & I_{\ell,t,dl,s} \geq T_{\text{crit},t}
\end{cases}
\] (13)

\[
T_{\text{safe},t} = T_{\ell} + \text{Cap} \cdot \sum_{t=1}^{t} \gamma_{\ell}^{t}
\]

Similar to the voltage limit, an overall satisfaction value is considered for each feeder, as follows:

\[
\mu_{I_{\ell,t}} = \frac{1}{8760} \sum_{dl=1}^{N_{dl}} \sum_{s=1}^{N_{s}} \text{prob}^{c}_s \cdot \tau_{dl} \cdot \mu_{I_{\ell,t},dl,s}
\] (14)

An index is needed to provide information about the overall performance of the system regarding the thermal limits. The average value of \(\mu_{I_{\ell,t}}\) over all feeders of the network can provide such information as follows:

\[
\mu_{I_{\ell}} = \frac{\sum_{\ell=1}^{N_{\ell}} \mu_{I_{\ell,t}}}{N_{\ell}}
\] (15)

For substation capacity constraint, also, the same philosophy holds, as follows:

\[
\mu_{S_{\text{grid},t},dl,s} = \begin{cases} 
1 & S_{\text{grid},t,dl,s} \leq S_{\text{safe},t} \\
\frac{S_{\text{grid},t,dl,s} - S_{\text{crit},t}}{S_{\text{safe},t} - S_{\text{crit},t}} & S_{\text{safe},t} \leq S_{\text{grid},t,dl,s} \leq S_{\text{crit},t} \\
0 & S_{\text{grid},t,dl,s} \geq S_{\text{crit},t}
\end{cases}
\] (16)

\[
\mu_{S_{\text{grid},t}} = \frac{1}{8760} \sum_{dl=1}^{N_{dl}} \sum_{s=1}^{N_{s}} \text{prob}^{c}_s \cdot \tau_{dl} \cdot \mu_{S_{\text{grid},t},dl,s}
\]

**D. Objective functions**

The proposed model minimizes three objective functions, namely, technical dissatisfaction, total costs and total emissions of the planning problem (see Appendix-I), as follows:

\[
\min \{ OF_1, OF_2, OF_3 \}
\]

subject to:

\[
(2) \rightarrow (16)
\]
The objective functions are formulated next.

1) Technical dissatisfaction: The first objective function to be minimized is dissatisfaction of technical constraints. The technical dissatisfaction, denoted by $TD_t$, is defined as the maximum dissatisfaction of all technical constraints as follows:

$$TD_t = 1 - \min \left\{ \mu^V_t, \mu^I_t, \mu^{grid}_t \right\}$$  \hspace{1cm} (17)

The objective function to be minimized is proposed here as the multiplication of maximum and average value of yearly technical dissatisfaction over planning horizon as:

$$OF_1 = w_{\text{avg}} \cdot \sum_{t=1}^{T} \frac{TD_t}{T} + w_{\text{sev}} \cdot \left(1 - \min_{t,dl,s,[\mu^{grid}_{t,dl,s}, \mu^V_{i,t,dl,s}, \mu^I_{\ell,t,dl,s}]} \right)$$  \hspace{1cm} (18)

By minimizing the $OF_1$, the algorithm tries to simultaneously improve the overall satisfaction of the network, represented by $\sum_{t=1}^{T} \frac{TD_t}{T}$, and the severity of technical dissatisfaction over the planning horizon, represented by the second term. In (18) the values of $w_{\text{sev}}$ and $w_{\text{avg}}$ are the weighting factor representing the importance of severity of technical dissatisfaction and the average dissatisfaction of technical constraints. If $w_{\text{sev}}$ is chosen much more bigger than $w_{\text{avg}}$, then the algorithm tries to find solutions which fully satisfy the technical constraints. On the other hand, if $w_{\text{avg}}$ is bigger than $w_{\text{sev}}$, then the technical satisfactions of the solutions are more relaxed.

2) Total costs: The second objective function, i.e., $OF_2$, to be minimized is the total costs which includes the cost of electricity purchased from the grid, the investment/operating costs of the DG units. The cost of energy procurement from the grid is calculated as:

$$TGC = \sum_{t=1}^{T} \sum_{dl=1}^{N_{dl}} \sum_{s=1}^{N_s} prob^s_s \cdot EP^\lambda_{t,s} \cdot P^{grid}_{t,s} \cdot \tau_{dl} \cdot \frac{1}{(1 + d)^t}$$  \hspace{1cm} (19)

Investment costs of the DG units can be calculated as:

$$DGIC = \sum_{t=1}^{T} \sum_{i=1}^{N_i} \sum_{dg=1}^{N_{dg}} \sum_{i,t,s=1}^{s} e^{i,t,s}_{dg} \cdot IC_{dg} \cdot \frac{1}{(1 + d)^t}$$  \hspace{1cm} (20)

The operating costs of the DG units can be calculated as:

$$DGOC = \sum_{t=1}^{T} \sum_{i=1}^{N_i} \sum_{dl=1}^{N_{dl}} \sum_{dg=1}^{N_{dg}} prob^s_s \cdot \tau_{dl} \cdot OC_{dg} \cdot \frac{1}{(1 + d)^t}$$  \hspace{1cm} (21)
The reinforcement cost of the distribution network is the sum of all costs paid for investment and operation of new feeders and transformers. The total feeder reinforcement cost, i.e. FC, and substation reinforcement cost, i.e. SC, are calculated as follows:

\[
FC = \sum_{t=1}^{T} \sum_{\ell=1}^{N_{\ell}} C_{\ell} . d_{\ell} . \cdot \gamma_{t}^{\ell} \cdot \frac{1}{(1 + d)^{t}}
\]

\[
SC = \sum_{t=1}^{T} C_{tr} . \cdot \psi_{t}^{tr} . \frac{1}{(1 + d)^{t}}
\]

Where, FC and SC are the total feeder and substation reinforcement cost, respectively. \(C_{\ell}, C_{tr}\) are the cost of each feeder and transformer, respectively.

Thus, \(OF_2\) is defined as:

\[
OF_2 = DGIC + DGOC + TGC + FC + SC
\]  

3) Total emission: The third objective function, i.e., \(OF_3\), is the total \(CO_2\) produced by the DG units and the main grid. \(OF_3\) can be formulated as:

\[
OF_3 = \sum_{t=1}^{T} \sum_{dl=1}^{N_{dl}} \sum_{s=1}^{N_{s}} prob_{s}^{dl} \cdot \tau_{dl} \cdot [E_{grid} \cdot P_{grid}^{t,s} + \sum_{i=1}^{N_{b}} \sum_{dg} E_{dg} \cdot P_{i,t,s}^{dg}]
\]

III. PROPOSED SOLUTION ALGORITHM

To solve the dynamic multi-objective DG planning problem formulated in section II, a two-stage algorithm is proposed in this section.

A. Proposed Binary PSO

The PSO algorithm is a population-based search technique proposed first by Kennedy and Eberhart in 1995 [21]. The basic idea of PSO is that each particle uses the swarm’s best experience as well as its own best experience in finding food. The PSO algorithm starts with a population of particles with random positions in the search space. Each particle is a solution of the problem and has a fitness value. The fitness is evaluated and is to be optimized. A velocity is defined which directs each particle’s position and gets updated in each iteration. Particles gradually move toward the optima due to their best position they have ever experienced and the best solution which group has experienced. The velocity of a particle is updated due to three
factors: the past velocity of the particle, the best position particle has experienced so far and the best position the entire swarm has experienced so far. In each iteration, every particle modifies its direction by its updated velocity affected by the three factors mentioned above. Mathematically the modification process may be expressed as follows:

\[
X_{p,iter+1}^{iter} = X_{p,iter}^{iter} + V_{p,iter}^{iter}
\]

\[
V_{p,iter+1}^{iter} = V_{p,iter}^{iter} + \text{Rand}_1 \cdot (X_{p,Best}^{iter} - X_{p,iter}^{iter})
\]

\[
+ \text{Rand}_2 \cdot (X_{g,Best}^{iter} - X_{p,iter}^{iter})
\]

\[p = 1, 2, \cdots, N_p\]

\(X_{p,iter}^{iter}\) is a particle which represents a potential solution to the optimization problem. In multi-dimensional problems like DG planning problem, each particle, \(X_{p,iter}^{iter}\) is a vector containing the decision variables and \(V_{p,iter}^{iter}\) is the velocity of particle \(p\), respectively. \(X_{p,Best}^{iter}\) is the best personal position of particle \(p\) has had up to now. Similarly \(X_{g,Best}^{iter}\) is the best global position which the entire particles have had up to now. The concept of direction modification in PSO implies that the direction of particle \(p\) is influenced by its present velocity, its best position up to now \(X_{p,Best}^{iter}\), and the best position of the whole particles up to now \(X_{g,Best}^{iter}\). In the context of multi-objective optimization (see Appendix-I for more details), it is needed that the population be directed toward the Pareto optimal front considering two important aspects: getting closer to Pareto optimal front and maintaining the diversity among the solutions [22]. To do so, a pseudo fitness value is assigned to each solution, referred to as affinity factor \(SF_n\), as follows:

\[
SF_n = w_1 \cdot FN_n^{-1} + w_2 \cdot GD_n
\]

The first term in (26) guides the population toward the Pareto optimal front since the solutions which belong to lower fronts get higher affinity (fitness). The second term insures the diversity among the solutions. In order to calculate the global diversity of the \(n^{th}\) solution, i.e. \(GD_n\), a local diversity factor, i.e. \(LD_n^k\), is defined for each objective function [22]. For every objective function \(k\), the solutions are sorted and the difference between the maximum and minimum values is calculated as:

\[
MD_k = \max_n(f_k(X_n)) - \min_n(f_k(X_n))
\]

\[n = 1, 2, \cdots, N_p\]
Since the solutions are sorted, the first and the last ones are the maximum and minimum, respectively. The local diversity of each of the other solutions is its average distance to its neighbors, as follows:

\[
LD_k^n = \frac{|f_k(X_n) - f_k(X_{n+1})| + |f_k(X_n) - f_k(X_{n-1})|}{2MD_k}
\]

For the first and the last solutions, local diversity can be calculated as:

\[
LD_{N_p}^1 = LD_1^k = max(LD_k^n)
\]

The global diversity factor for each solution is then calculated as the average of its local diversities as follows:

\[
GD_n = \frac{1}{N_o} \sum_{k=1}^{N_o} \frac{LD_k^n}{N_o}
\]

In initial iterations, a small number of solutions belong to the first Pareto front, so getting closer to Pareto optimal front is more important than maintaining the diversity among them. It is necessary to enable the algorithm in distinguishing between the solutions in different Pareto fronts, \(w_1\) and \(w_2\) in (26) are adaptively selected which guarantees that the solution belonging to a lower Pareto front has a bigger affinity factor than a solution belonging to an upper front level (\(w_1\) is bigger than \(w_2\) in the initial iterations) and when most of the solutions are in the pareto optimal front, \(w_2\) is chosen bigger than \(w_2\) to maintain the diversify among the solutions.

B. The Proposed two-stage solution algorithm

The solution algorithm proposed here consists of two stages. In the first stage, the solutions which form the Pareto optimal front are found and in the second stage, the best solution is selected considering the planner’s preferences. Both stages are described as follows:

1) Stage I (finding the Pareto optimal front): The PSO algorithm proposed in section III-A is used to find the Pareto optimal front. To do so, each particle is a vector containing the decision variables coded in binary format (to show the investment decisions) investment decision of DG units, the bus on which a DG units is to be installed, the year of investment and their generated power and for all available DG technologies. Steps of the first stage of the solution algorithm are as follows:
step. 1. Generate an initial random particles.
step. 2. If the stopping criterion is met, go to step \( k \), else, continue.
step. 3. Calculate \( OF_1, OF_2, OF_3 \) for each particle.
step. 4. Calculate the fitness using (26) for each particle.
step. 5. Calculate the \( V_{p}^{iter} \)
step. 6. Calculate the \( X_{g,Best}^{iter}, X_{p,Best}^{iter} \)
step. 7. Construct a new population by moving the particles.
step. 8. Return to step (2).
step. 9. End.

The flowchart of the first stage of the proposed method is depicted in Fig.3.

2) Stage II (Selecting ‘the best’ solution): The decision maker (planner) needs a tool to select the final solution among the existing solutions of Pareto front. In this paper a fuzzy satisfying method is used for this purpose. The concept of this method is as follows: for each solution in the Pareto optimal front, \( X_n \), a membership function is defined as \( \mu_{f_k}(X_n) \). This value, which varies between 0 to 1, shows the level of which \( X_n \) belongs to the set that minimizes the objective function \( f_k \). A linear membership function is used here for all objective functions as follows:

\[
\mu_{f_k}(X) = \begin{cases} 
0 & f_k(X) > f_k^{max} \\
\frac{f_k^{max} - f_k(X)}{f_k^{max} - f_k^{min}} & f_k^{min} \leq f_k(X) \leq f_k^{max} \\
1 & f_k(X) < f_k^{min} 
\end{cases} \tag{31}
\]

A conservative decision maker tries to maximize minimum satisfaction among all objectives or minimize the maximum dissatisfaction [23]. The final solution can then be found as:

\[
\begin{align*}
N_{p}^{max} &= \max_{n=1}^{N_o} \min_{k=1}^{N_o} \mu_{f_k}(X_n) \\
N_{p}^{max} &= \frac{N_o}{\max_{n=1}^{N_o} \min_{k=1}^{N_o} \mu_{f_k}(X_n)} \tag{32}
\end{align*}
\]

IV. SIMULATION RESULTS

The proposed methodology is applied to a realistic 201-node 10 kV distribution system which is shown in Fig.4. The technical data of this network can be found in [24]. Three DG technology options, namely, Micro Turbine (MT), Wind Turbine (WT), Gas Turbine (GT) are considered here. It is also assumed that all buses are candidate for DG investment and more than one DG can be installed in a specific bus. The stopping criterion is reaching to a predefined maximum number of iterations. The Rayleigh parameter of the wind speed in each wind farm has been
assumed to be $c = 8.78$ and the other characteristics of wind turbine are given in Table II. Using the technique described in [19], the PDF of wind speed is divided into twelve states as given in Table III. The forecasted values of demand and price level factors are given in Table IV [25]. The $\sigma$ value of each demand level is assumed to be 1% of its forecasted value. Other simulation assumptions and characteristics of the DG units [26], [27] are presented in Table V and Table VI respectively. The presented solution algorithm was implemented in MATLAB. The number of demand levels, i.e. $N_{dl}$ is assumed to be 24 and the duration of each demand level is 365hr. In (18) the values of $w_{avg}$ and $w_{sev}$ are assumed to be 0.8 and 0.2, respectively. Solving the (6) gives $7 \times 7 \times 12 = 588$ states for each demand level. It is clear that solving the evaluation process for all of these states (for all demand levels) imposes a heavy computational burden. In order to resolve this problem, a scenario reduction technique proposed in [28] has been used to reduce the number of states (see Appendix-II for more details). The scenario set is reduced into 110 states (this is chosen based on trial and error) using the described technique. The formulated problem is solved using the proposed two-stage algorithm and 80 non-inferior solutions are found. The planner can choose the best solution based on the planning criteria, as further discussed next. The Pareto optimal front of the search space, found in the first stage, is depicted in Fig.5. The variation ranges of all objective functions are given in Table VII. In the second stage, the planner can choose the most preferred solution using the fuzzy satisfaction method introduced in section III-B. The final solution is solution #68. The various costs related to the selected solution are given in Table VIII. The investment plan of this solution is described in Table IX. It would be interesting to know how much accuracy is lost if the scenarios are reduced. The final solution (which was already found using the $N_s = 110$) is reevaluated using various values of $N_s$. The exact values of this solution is obtained if all scenarios are considered. This value is taken as a reference for comparing the results obtained by various number of reduced scenarios. The computation error due to scenario reduction is depicted versus the number of reduced scenarios, i.e $N_s$, Fig.6. This figure shows that if the number of scenarios is chosen to be greater than 93, then the accuracy is acceptable and the fluctuation is highly reduced and the error will be less than 0.005%.

In order to analyze the performance of the proposed binary PSO methodology, it is compared with an standard real coded PSO as described in [29]. For this purpose, the problem is solved just for two objectives namely, technical satisfaction and cost (this is done just because the
comparison is much more easier). The initial solution (random solution) and the Pareto optimal fronts found by each PSO method are depicted in Fig. 7. Both algorithms are run for 100 iterations. It is obvious that the Pareto front obtained by binary PSO dominates the solutions of the read coded algorithm. This means for each solution in Pareto front of real coded PSO there exists at least one solution in the BPSO front that gives lower cost and technical dissatisfaction simultaneously.

The model can be directly used in power market model in which the DNO is authorized for DG investment. However, in power market models where the DG investment is done by independent investors instead of DNO, It can be easily modified to be used in such regulatory frameworks. The decisions related to investment and operating of DG units are made by private entities. In this case, the values of $\xi_{i,t}^{dg}$ are determined by DG owners. The decision variables of DNO are $\gamma_{i,t}^{c}$ and $\psi_{i,t}^{f}$ (network investment options). The provided information would also be useful as a technical, economical and environmental signal for regulators. It can be used for regulating the incentives to encourage the private section to invest in what DG technology and where to be more beneficial.

V. Conclusion

This paper presents a dynamic multi-objective model for DG planning problem and a binary PSO based method to solve the formulated problem. The proposed two-step algorithm finds the non-dominated solutions by simultaneous minimization of technical dissatisfaction, costs and emissions in the first stage and uses a fuzzy satisfying method to select the best solution from the candidate set in the second stage. The new planning model is applied to a realistic distribution network and its flexibility is demonstrated through different case studies. The solution set provides the planner with an insight into the problem and enables him to choose the best solution according to planning preferences.
APPENDIX-I: PARETO OPTIMALITY CONCEPT

In general, in a multi-objective optimization problem, more than one objective function needs to be simultaneously optimized as follows:

\[
\min F(X) = [f_1(X), \ldots, f_{N_O}(X)]
\]  

**Subject to:**

\[
\{G(X) = 0, H(X) \leq 0\}
\]

\[X = [x_1, \ldots, x_m]\]

Suppose \(X_1\) and \(X_2\) belong to the solution space. \(X_1\) dominates \(X_2\) if:

\[
\forall k \in \{1 \ldots N_O\} \ f_k(X_1) \leq f_k(X_2)
\]

\[
\exists k' \in \{1 \ldots N_O\} \ f_{k'}(X_1) < f_{k'}(X_2)
\]

Any solution which is not dominated by any other is called to belong to a Pareto front which is referred to as the first Pareto front or optimal front or non-dominated front.

APPENDIX-II: SCENARIO REDUCTION TECHNIQUE

The purpose of scenario reduction is selection of a set, i.e. \(\Omega_S\), with the cardinality of \(N_{\Omega_S}\), from the original set, i.e. \(\Omega_J\) [30]. This procedure should be done in a way that makes a trade off between the loss of the information and decreasing the computational burden [31]. The scenario reduction technique used in this paper is described as the following steps [28]:

step. 1 Construct the matrix containing the distance between each pair of scenarios \(c(w, \hat{w})\)

step. 2 Select the first scenario \(w_1\) as follows:

\[
w_1 = \arg \left\{ \min_{w' \in \Omega_J} \sum_{w \in \Omega_J} \pi_w c(w, w') \right\}
\]

\[\Omega_S = \{w_1\}, \Omega_J = \Omega_J - \Omega_S\]

step. 3 Select the next scenario to be added to \(\Omega_S\) as follows:

\[
w_n = \arg \left\{ \min_{w' \in \Omega_J} \sum_{w \in \Omega_J - \{w'\}} \pi_w \min_{w'' \in \Omega_S \cup \{w\}} c(w, w'') \right\}
\]

\[\Omega_S = \Omega_S \cup \{w_n\}, \Omega_J = \Omega_J - \Omega_S\]
step. 4 If the number of selected set is sufficient then end and go to step 2 ; else continue.

step. 5 The probabilities of each non-selected scenario will be added to its closest scenario in the selected set.

step. 6 End.

REFERENCES


List of figure captions:

1) Fig.1 Demand and price level factor uncertainty modeling
2) Fig.2 The idealized power curve of a wind turbine
3) Fig.3 Flowchart of the first stage of the proposed method
4) Fig.4 Single-Line Diagram of the real system under study
5) Fig.5 Pareto optimal front found by the algorithm
6) Fig.6 Sensitivity analysis of the computation accuracy versus the number of scenarios
7) Fig.7 Comparison between Binary PSO and real coded classic PSO
### TABLE I

DG planning methods

<table>
<thead>
<tr>
<th>Reference</th>
<th>Single/Multi objective</th>
<th>Static/ Dynamic</th>
<th>Uncertainty handling</th>
<th>Network reinforcement</th>
<th>Method</th>
</tr>
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<tbody>
<tr>
<td>El-Khattam et al. [32]</td>
<td>S</td>
<td>S</td>
<td>N</td>
<td>Y</td>
<td>Classic MINLP</td>
</tr>
<tr>
<td>Jabr et al. [33]</td>
<td>S</td>
<td>S</td>
<td>N</td>
<td>Y (not exact)</td>
<td>Ordinal optimization</td>
</tr>
<tr>
<td>El-Khattam et al. [34]</td>
<td>S</td>
<td>S</td>
<td>N</td>
<td>N</td>
<td>Classic MINLP</td>
</tr>
<tr>
<td>Wang et al. [35]</td>
<td>S</td>
<td>D</td>
<td>N</td>
<td>Y</td>
<td>Greedy heuristic</td>
</tr>
<tr>
<td>Kumar et al. [36]</td>
<td>S</td>
<td>S</td>
<td>N</td>
<td>N</td>
<td>Classic MINLP</td>
</tr>
<tr>
<td>Wong et al. [37]</td>
<td>S</td>
<td>D</td>
<td>N</td>
<td>Y</td>
<td>Classic MINLP</td>
</tr>
<tr>
<td>Zangeneh et al. [38]</td>
<td>M</td>
<td>S</td>
<td>N</td>
<td>N</td>
<td>Normal boundary intersection</td>
</tr>
<tr>
<td>Haghifam et al. [17]</td>
<td>M</td>
<td>S</td>
<td>Y</td>
<td>N</td>
<td>Heuristic NSGA-II</td>
</tr>
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<td>Atwa et al. [19]</td>
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<td>S</td>
<td>Y</td>
<td>N</td>
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<td>S</td>
<td>N</td>
<td>N</td>
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</tr>
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<td>Harrison et al. [41]</td>
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<td>N</td>
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<td>$\epsilon$-constrained technique</td>
</tr>
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<td>Soroudi et al. [18]</td>
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<td>D</td>
<td>N</td>
<td>Y (exact)</td>
<td>Immune algorithm</td>
</tr>
<tr>
<td>proposed</td>
<td>M</td>
<td>D</td>
<td>Y</td>
<td>Y (exact)</td>
<td>Heuristic IPSO</td>
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### TABLE II

THE TECHNICAL CHARACTERISTICS OF WIND TURBINES

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<th>$v_{\text{cut in}}^{\text{r}}$</th>
<th>$v_{\text{rated}}^{\text{r}}$</th>
<th>$v_{\text{cut out}}^{\text{r}}$</th>
<th>$P_{\text{wea}}^{\text{r}}$</th>
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<td>(m/s)</td>
<td>(m/s)</td>
<td>(m/s)</td>
<td>(MW)</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
<td>25</td>
<td>0.5</td>
</tr>
<tr>
<td>State</td>
<td>$w p_s$ (%)</td>
<td>$Prob_{w s}^*$</td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>-------------</td>
<td>----------------</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0.1105</td>
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</tr>
<tr>
<td>2</td>
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<td>3</td>
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<td>0.0973</td>
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</tr>
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</tr>
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<td>65</td>
<td>0.0764</td>
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</tr>
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<td>9</td>
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<td>0.0652</td>
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<td>11</td>
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<td>12</td>
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<td></td>
</tr>
<tr>
<td>( dl )</td>
<td>( D_{at} )</td>
<td>( \lambda_{at} )</td>
<td></td>
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<td>------</td>
<td>------</td>
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<td></td>
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<td>15</td>
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<td>18</td>
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<td>20</td>
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### TABLE V
Data used in the study

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<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
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<tr>
<td>$T$</td>
<td>year</td>
<td>8</td>
</tr>
<tr>
<td>$S_{se,t=0}^{cr}$</td>
<td>MVA</td>
<td>32</td>
</tr>
<tr>
<td>$S_{crit,t=0}^{cr}$</td>
<td>MVA</td>
<td>40</td>
</tr>
<tr>
<td>$E_{grid}$</td>
<td>kgCO\textsubscript{2}/MWh</td>
<td>632</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$$/MWh.</td>
<td>60</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>%</td>
<td>1</td>
</tr>
<tr>
<td>$d$</td>
<td>%</td>
<td>12</td>
</tr>
<tr>
<td>$V_{safe}^{max}$</td>
<td>Pu</td>
<td>1.05</td>
</tr>
<tr>
<td>$V_{crit}^{max}$</td>
<td>Pu</td>
<td>(1+5%)</td>
</tr>
<tr>
<td>$V_{safe}^{min}$</td>
<td>Pu</td>
<td>0.95</td>
</tr>
<tr>
<td>$V_{crit}^{min}$</td>
<td>Pu</td>
<td>(1-5%)</td>
</tr>
<tr>
<td>$I_{s,t}$</td>
<td>A</td>
<td>0.9 × $I_{crit,t}$</td>
</tr>
<tr>
<td>$N_p$</td>
<td></td>
<td>80</td>
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</table>

Maximum iteration 1000

### TABLE VI
Characteristics of the DG units

<table>
<thead>
<tr>
<th>DG Technology</th>
<th>Size</th>
<th>Emission $kgCO_2/MWh$</th>
<th>IC $k$/MVA</th>
<th>OC $$/MWh$</th>
<th>cos$\phi$</th>
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<tr>
<td>Micro Turbine</td>
<td>0.5</td>
<td>502</td>
<td>1485</td>
<td>75</td>
<td>0.9</td>
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<td>Gas Turbine</td>
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<td>365</td>
<td>1030</td>
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<td>Wind turbine</td>
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<td>0</td>
<td>1225</td>
<td>45</td>
<td>1</td>
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### TABLE VII
Variation range of objective function for all solution in Pareto optimal front

<table>
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<tr>
<th>OF\textsubscript{1}</th>
<th>OF\textsubscript{2} ($)</th>
<th>OF\textsubscript{3} (Ton CO\textsubscript{2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_k^{min}$</td>
<td>0.0026</td>
<td>$9.7208 \times 10^7$</td>
</tr>
<tr>
<td>$f_k^{max}$</td>
<td>0.0689</td>
<td>$2.4936 \times 10^9$</td>
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</tbody>
</table>

$9.7208 \times 10^7$
TABLE VIII
The investment plan obtained for the final solution

<table>
<thead>
<tr>
<th>DGtech</th>
<th>$c_{d,t}^{d}$</th>
<th>Year</th>
<th>Bus</th>
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<tbody>
<tr>
<td>Micro Turbine</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>201</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>39,114,26</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>164,201</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>39</td>
<td></td>
</tr>
<tr>
<td>Gas Turbine</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>152,102</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>14,177</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>177,102</td>
<td></td>
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<tr>
<td>1</td>
<td>4</td>
<td>76</td>
<td></td>
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<tr>
<td>1</td>
<td>5</td>
<td>201</td>
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<tr>
<td>1</td>
<td>7</td>
<td>14</td>
<td></td>
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<td>8</td>
<td>201</td>
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<tr>
<td>Wind Turbine</td>
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<td></td>
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<td>1</td>
<td>1</td>
<td>26,39,64,89,114,139</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>201</td>
<td></td>
</tr>
</tbody>
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TABLE IX
Investment/operating cost in final solution (M$)

<table>
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<tr>
<th>Year</th>
<th>GC</th>
<th>IC</th>
<th>sub</th>
<th>feeder</th>
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<tbody>
<tr>
<td>1</td>
<td>9.624959</td>
<td>18.4675</td>
<td>0</td>
<td>0</td>
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<tr>
<td>2</td>
<td>9.670223</td>
<td>2.485</td>
<td>0</td>
<td>34.5</td>
</tr>
<tr>
<td>3</td>
<td>9.662216</td>
<td>1.7425</td>
<td>0</td>
<td>89.25</td>
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<tr>
<td>4</td>
<td>9.680735</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>9.705028</td>
<td>0.5</td>
<td>0</td>
<td>73.5</td>
</tr>
<tr>
<td>6</td>
<td>9.813719</td>
<td>0</td>
<td>0</td>
<td>25.2</td>
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<tr>
<td>7</td>
<td>9.835628</td>
<td>0.5</td>
<td>0</td>
<td>90</td>
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<tr>
<td>8</td>
<td>9.861077</td>
<td>0.5</td>
<td>0.2</td>
<td>78</td>
</tr>
</tbody>
</table>

October 10, 2011 DRAFT
Fig. 1. Demand and price level factor uncertainty modeling
Fig. 2. The idealized power curve of a wind turbine

Fig. 3. Flowchart of the first stage of the proposed method
Fig. 4. Single-Line Diagram of the real system under study

Fig. 5. Pareto optimal front found by the algorithm
Fig. 6. Sensitivity analysis of the computation accuracy versus the number of scenarios

Fig. 7. Comparison between Binary PSO and real coded classic PSO