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Abstract

This paper examines the use of the cross-entropy (CE) method to estimate the structural parameters of a plate structure, given data from a simulated non-destructive static loading test. Finite element models of plates are created, with properties close to that of a bridge of similar dimensions. Damage is introduced to the models by local reductions in the longitudinal Young's modulus (E_x). In practice, reduced values of E_x may result from material irregularities, poor construction methods and structural degradation due to weathering and/or impact. By assembling combinations of E_x values, the CE method searches the solution space of possible combinations of E_x values. The location and severity of the damage is varied to test the ability of the algorithm to identify different damage events.

Keywords: cross-entropy method, stiffness estimation, damage detection, structural health monitoring, finite element modelling, plates.

1 Introduction

In recent years much research has been carried out in the field of structural health monitoring to find new ways of ensuring the integrity and reliability of machines, buildings and built infrastructure. The ability to detect the location and severity of damage to structures in an infrastructure network allows maintenance authorities to better allocate resources to perform further investigations on structures believed to be damaged.

This paper examines the use of the CE method to estimate the structural parameters of a plate structure, given data from a simulated non-destructive test based on static loading. The method is demonstrated for a plate-type structure, but the same procedure can be extended to other structural forms. The method performs combinatorial optimisation to estimate stiffness given the plate's deflection response to static loading. This loading case is similar to the proof load test which has been

used in practice to assess bridge structures. In the field, deflection values can be measured by linear variable differential transducers or laser-based systems. Alternatively, fibre optic sensors may be incorporated into a structure at the time of construction, allowing structural deflections and rotations to be monitored continuously.

Many researchers have attempted to assess structural integrity by analysing dynamic signals [1, 2]. Static load testing has the advantage of giving relatively low levels of noise compared to dynamic testing. Authors have various approaches to analysing the output from static load testing for structural identification. Neural networks have been trained to estimate the location and severity of damage in plate structures mounted with optical sensing fibre Bragg gratings [3]. Once trained, neural networks operate quickly to recognise patterns presented during training, and make use of underlying trends in the data to generalise beyond the training patterns. To perform this however, neural networks must be presented with a training set that adequately represents the complexity of the problem. For complex systems, adequate enumeration of the solution space is a formidable task. The CE method presented here does not require training, or the creation of a training set.

A non-quadratic optimisation procedure has been developed to identify damage to Euler-Bernoulli beams modelled as Dirac's delta distributions in flexural stiffness [4]. Static load testing has been used to perform model updating on a specimen frame structure [5]. The authors expressed deflection and strain error functions as nonlinear algebraic functions of stiffness parameters, which in turn were solved by Gauss-Newton and gradient methods. Genetic algorithm, an alternative optimisation technique to the CE method based on evolutionary theory, has been used to identify the location and severity of damage in structures [6]. Damage was modelled by reducing the cross-sectional area of up to three damaged truss members. The authors were able to estimate closely the location and severity of damage for trusses. Static load testing has been used to detect damage in plane bow-string truss and plane frame structures by solving a constrained optimisation problem for updated structural parameters [7].

The CE method presented here develops on previous work which applied CE to damage detection in 1-dimensional, simply-supported beam structures, with 4 degrees of freedom per element [8]. The current work applies the CE method to 2-dimensional plate models, with 16 degrees of freedom per element. The structures in this paper take into account transverse stiffness. Each plate model requires greater computational effort than the 1-dimensional beam, and the additional elements give rise to more parameters to be estimated by CE, making for a significant increase in computational effort. In this paper, the CE algorithm systematically searches the solution space of possible stiffness combinations, to give a plate with deflection properties close to that of the plate under investigation. If the resulting model shows irregularities in the structure, engineering judgement must be used to decide whether the structure is in need of further investigation and/or remedial work. In this way parameter estimation can play a useful role in structural health monitoring. However, the estimation of the stiffness characteristics of an existing structure is an inverse problem with many possible solutions. Variations in Young's modulus due to inconsistencies in the structural material or as a result of cracking, ageing, loading

or aggressive environmental conditions, may lead to significant differences in the stiffness distribution not accounted for in design. When assessing an existing structure, it is essential to quantify these discrepancies with respect to the original design, and the current loading conditions, to ensure the structure remains safe.

To investigate the versatility of the system, a number of damage events are modelled by varying the severity and location of damage. Various parameters of the CE method are altered to investigate the sensitivity of the system. The effects of varying the number of measurement points on the plate and having more than one weakened section in the structure are discussed. Although it is reasonable to expect relatively low levels of noise with static load testing, measurements may still be corrupted to some extent, due to a faulty sensor or poor installation. For this reason, simulations are also carried out in the presence of noisy data.

2 Cross-Entropy and Damage Detection

The CE method was first proposed by Rubinstein for rare event simulation and combinatorial optimisation [9]. The CE method presented here is an iterative process. Throughout the process the system represents the stiffness of each plate element not with a single value, but with a distribution of values. At each iteration the system creates a number of combinations of stiffness values from these elemental distributions. As the system progresses, the properties of the elemental distributions are altered based on the stiffness patterns giving deflection response closest to that of the structure under investigation.

2.1 Plate Model

At each iteration, the CE method creates finite element (FE) plate models and compares the deflection performance of each to that of the plate under investigation. The FE plates are composed of 2-dimensional orthotropic elements, each with 4 nodes and 4 degrees of freedom per node. The global stiffness matrix is composed of $[16 \times 16]$ elemental stiffness matrices [10]. Similar to a standard Kirchhoff element, the degrees of freedom at each node describe vertical displacement, curvature about the x-axis and curvature about the y-axis [11]. A fourth degree of freedom, ‘nodal twist’ is added to give a conforming element. The forcing vector describes direct vertical force, bending moment about the x- and y-axes, and a twisting force at each node. Shape functions are used to distribute forces applied anywhere in the system to the appropriate nodes.

All plates in this paper are assumed to be of span 16m, width 12m and depth 0.8m. Longitudinal Young’s modulus for a healthy element is taken as $E_{x0} = 3.5 \times 10^{10} \text{N/m}^2$. The value of longitudinal Young’s modulus for damaged elements, E_{xd} , varies. All plates are orthotropic, with transverse Young’s modulus, $E_y = 3.22 \times 10^{10} \text{N/m}^2$. Poisson’s ratio is 0.2, and shear modulus is $1.4 \times 10^{10} \text{N/m}^2$. Static loading of 50kN is applied at 4 points, corresponding to the points of contact of a 2-axle truck of weight 200kN located at midspan. The truck axle spacing is 4m and the distance between wheels of an axle is 2m.

Damage is introduced to the system by local reductions in longitudinal Young's modulus, E_x . Support conditions in all simulations are simply supported. Figure 1 shows the stiffness pattern for damage event 'DEA', with stiffness reduced to $E_{xd} = 0.7E_{x0}$ for 6 of the 48 plate elements.

Figure 1: E_x values for damage event 'DEA'

Figure 2 shows the deflection pattern resulting from subjecting a plate, with stiffness distribution corresponding to 'DEA', to the simulated static loading test. For all simulations, the reference deflection pattern is found from a fine mesh with 768 elements, each of area 0.25m^2 . The CE algorithm formulates a plate of similar stiffness properties with 48 elements, each of area 4m^2 .

Figure 2: Deflection pattern for damage event 'DEA'

2.2 The Cross-entropy Algorithm

Throughout simulation, the CE method represents the E_x value of each element not with a single value, but with a distribution of values. The distribution for element n , at iteration k , is assumed to follow a normal distribution defined by mean μ_n^k and standard deviation σ_n^k . The system is initialised with μ_n^1 values reasonably close to the actual E_x value for a healthy element and σ_n^1 values large enough to give a broad range of possible E_x values. At iteration k , N selections are made from each elemental distribution giving N combinations of elemental E_x values. Each set of randomly chosen E_x values is combined with the known geometry of the plate elements to give N trial plates (TPs). The deflection response of each of these TPs to the static loading test is found by FE modelling. The CE method estimates plate stiffness given deflection readings at D locations on the plate. The set of deflection readings measured from the specimen plate is denoted $\Delta^m = \{\delta_1^m, \delta_2^m, \dots, \delta_D^m\}$. The target deflection pattern for the algorithm is given by normalising Δ^m , to give $\{\bar{\delta}_1^m, \bar{\delta}_2^m, \dots, \bar{\delta}_D^m\}$, according to Equation (1). The deflection response of each TP found by FE modelling is denoted $\Delta^t = \{\delta_1^t, \delta_2^t, \dots, \delta_D^t\}$. The elements of Δ^t are normalised in the same way to give $\{\bar{\delta}_1^t, \bar{\delta}_2^t, \dots, \bar{\delta}_D^t\}$, as in Equation (2).

$$\bar{\delta}_i^m = \left(\frac{\max(\Delta^m) - \delta_i^m}{\max(\Delta^m) - \min(\Delta^m)} \right) \quad i = 1, \dots, D \quad (1)$$

$$\bar{\delta}_i^t = \left(\frac{\max(\Delta^m) - \delta_i^t}{\max(\Delta^m) - \min(\Delta^m)} \right) \quad i = 1, \dots, D \quad (2)$$

The algorithm progresses by choosing combinations of stiffness values that give deflection performance closest to that observed in the actual structure. For this purpose, TPs are ranked according to the error objective function, F , given by the sum of the squared errors over all measured degrees of freedom, as in Equation (3). The portion of TPs with lowest F value are known as the ‘elite set’. All TPs outside of the elite set are discarded, and the μ_n^{k+1} and σ_n^{k+1} values are derived only from the TPs of the elite set. In this paper, the ‘elite set’ is always composed of 10% of the TPs at any iteration.

$$F = \sum_{i=1}^D \left(\bar{\delta}_i^m - \bar{\delta}_i^t \right)^2 \quad (3)$$

The CE method performs a systematic search of the solution space, based on the properties of combinations formed by random selections from elemental distributions. As the search proceeds, it is expected that the mean of the elemental distributions will gravitate towards the actual E_x values, and that the standard deviations will decrease, giving narrower distributions. This narrowing of the

elemental distributions can lead to the system becoming trapped in local minima. By periodically increasing the standard deviation of all elements in the system, the CE method can escape local minima. This technique is known as ‘injection’. As the algorithm proceeds, the rate of change of the error objective function, F , is monitored. Once F changes by less than 5% over 10 iterations, the algorithm is said to have converged. At convergence, injection may be applied or the algorithm may be stopped. For the simulations in this paper, injection is applied 5 times before the system is stopped. The final output for each simulation is the mean of the elite set from the iteration giving lowest F . When injection is first applied the standard deviation is increased to its initial value. Thereafter the quantity of injection is reduced to allow the system to settle into a solution.

Figure 3 shows two separate measures of error at each iteration for an entire run of the CE algorithm. The dotted line shows the level of mean percentage error (MPE) for that iteration, that is the mean value of the percentage error between the target stiffness pattern and the mean of the elite set for that iteration. The solid line shows the development of the error objective function, F . The peaks in the solid line showing F values indicate the points at which injection was applied. The MPE graph does not show the same peaks as MPE is found only from the TPs of the elite set at each iteration, whereas F is calculated over the entire population of TPs.

Figure 3: Mean percentage error (MPE) and error objective function (F) for each iteration

It can be seen that both measures of error reduce significantly from their initial values. The system attempts to achieve a low value of output MSE by reducing the error objective function, F . Output for the system is taken at the iteration with lowest F value, in this case that is the final iteration. This graph shows the close relationship between stiffness estimation and the resulting F value on which the CE method presented here is based. It is noted that although the lowest MPE value occurs at iteration 379, the algorithm continues to reduce the error

objective function. This illustrates the difficulty in this inverse identification problem.

2.3 Relationship Between σ_n and Error

The CE method presented here represents the stiffness of element n at iteration k by a distribution of values, defined by mean, μ_n^k , and standard deviation, σ_n^k . The pattern of elemental standard deviation, throughout the plate reflects the extent to which the stiffness of each element influences plate deflection, and hence, the error objective function, F . Figure 4 shows the values of σ_n^k at iterations $k = 1, 100$ and 199 for simulation with ‘DEA’, $N = 10,000$ and $D = 49$. At iteration 1 the system is initialised and all elements show equal σ_n^1 values. As the algorithm progresses, σ_n^k values decrease. For this simulation the lowest value of F was found at iteration 199.

Figure 4: Standard deviation, σ_n , for each element: (a) at iteration 1, (b) at iteration 100 and (c) at iteration 199

It can be seen that σ_n^k tends to be lowest in elements close to midspan, and higher close to the supports. Elements close to midspan have the greatest impact on plate deflection response, and hence the system shows less variance in possible E_x values. The highest values of σ_n occur in the central elements of the sections at the supports. Elements at the free edge of the plate are surrounded by fewer elements than those at the centre, leading to reduced uncertainty in stiffness estimation at the plate edge. This same effect can be seen in estimating stiffness of edge elements in section 3.2.

3 Theoretical Testing

A variety of damage events are modelled and subject to a simulated proof loading test. The CE method is used to estimate the distribution of E_x values in the plate.

Various parameters of the CE method are altered to assess the sensitivity of the system. Output from various CE simulations may be compared by considering the output MPE, given by the mean of the errors between the stiffness values estimated by CE and the actual stiffness values in the structure under investigation, expressed as a percentage of the actual stiffness values. For all simulations it is assumed that the stiffness of the system is not known a priori, and so the system is initialised with distribution mean $\mu_n^1 = 4 \times 10^{10}$ (equal to $1.25 E_{x0}$) and standard deviation $\sigma_n^1 = 4 \times 10^9$.

3.1 Number of Trial Plates per Iteration

At each iteration, the CE method makes N random selections from the elemental stiffness distributions, and hence creates N combinations of E_x values. It is reasonable to expect that a greater value of N will perform a more thorough search of the solution space of possible stiffness combinations for a given number of iterations, however increasing the value of N also increases computational time. Figure 5 shows the error for various values of N , for a plate with damage event ‘DEA’. For each value of N , the CE algorithm is performed 3 times with all settings unchanged; the different output for each simulation is due to the randomness of the selections made throughout the process. For these simulations deflection readings are taken at all internal nodes and injection is applied 5 times. The resulting output MPE is labelled R1, R2 and R3. As the number of selections is increased, the mean error over 3 simulations is lower.

Figure 5: Output MPE for various values of the sample size N

3.2 Damage Event

The CE method is applied to damage events ‘DEA’, ‘DEB’ and ‘DEC’. In these simulations $N = 10,000$, injection is applied to the system 5 times and deflection readings are taken at all 49 internal nodes. ‘DEA’ is defined in Figure 1, with $E_{x0} = 3.5 \times 10^{10} \text{ N/m}^2$ and $E_{xd} = 0.7 E_{x0}$. ‘DEB’ shows stiffness reduction in the same location as ‘DEA’, but damage is less severe, with $E_{xd} = 0.8 E_{x0}$. Figures 6 and 7

show the target E_x values, compared with the output from CE for ‘DEA’ and ‘DEB’ respectively. In these figures, dark areas indicate that E_x has been overestimated by CE, white areas indicate E_x has been underestimated by CE. Where only a grey bar is shown, CE has estimated E_x to within $\pm 3\%$ of the actual value.

Figure 6: (a) Target stiffness values for ‘DEA’, (b) Stiffness estimated by CE for ‘DEA’

Figure 7: (a) Target stiffness values for ‘DEB’, (b) Stiffness estimated by CE for ‘DEB’

In estimating the stiffness of both ‘DEA’ and ‘DEB’, the algorithm shows the lowest stiffness values at the damaged locations. Estimation for E_x of healthy elements is a little erratic, with the greatest error occurring in elements at the supports. This is due to elements close to the supports having a relatively small impact on plate deflection. It is noted that some elements are identified to within $\pm 3\%$ of the actual value.

A new damage event ‘DEC’ is defined by a stiffness reduction at two separate locations, as in Figure 8(a). For both damage locations $E_{xd} = 0.7E_{x0}$. Again,

CE has shown lowest stiffness values in the correct elements, giving a clear indication of the location of both damaged areas. CE again gives greatest error close to the supports, on this occasion underestimating E_x at the left support adjacent to the damaged elements.

Figure 8: (a) Target stiffness values for 'DEC', (b) Stiffness estimated by CE for 'DEC'

It is noted that in many cases the stiffness of elements close to the longitudinal edge is estimated more closely than that of elements in the plate interior. Edge elements are surrounded by fewer elements than those in the interior, which reduces the level of uncertainty close to the edge. A similar trend is seen in the distribution of σ_n values (Section 2.3).

3.3 Initial Mean Values & Pre-simulation

The CE algorithm is initialised with assumed values for E_x . Previous work on the CE method for stiffness estimation indicated that the algorithm was sensitive to these initial values [8]. It was found that the system benefited from a brief "pre-simulation" to bring the initial mean values closer to the actual stiffness values. The current algorithm makes use of similar pre-simulation by artificially adjusting the elemental distribution means. Once convergence has occurred for the second time, the distribution mean of each element is given by the mean of the estimated stiffness value for that element, and all adjacent elements. This resetting of the distribution μ_n values moderates some of the extreme values in the plate, and may allow the algorithm to search previously unexplored regions of the solution space.

Assuming E_x for each element is a discrete random variable, with P possible E_x values, the number of possible combinations of stiffness values for a plate with L elements is given by P^L . This relationship indicates the difficulty inherent in the inverse problem of stiffness estimation given measured structural response. For the CE method presented here to be applied to more complex systems, the ability to reduce the required computational effort is desirable. For stiffness estimation in

plates, one approach is to perform a portion of the simulation with a relatively coarse plate mesh. Fewer plate elements allow the deflection pattern for each TP to be computed more easily. Having fewer parameters to estimate also allows the CE method to perform each iteration more quickly. This concept was applied the current problem by performing pre-simulation with a relatively coarse mesh of 30 elements, as opposed to the usual mesh of 48 elements. Although fewer elements could be used for this pre-simulation, too coarse a mesh could give over-simplified TPs, incapable of matching the behaviour of the plate under investigation.

Figure 9 shows the MPE for three runs with coarse mesh pre-simulation and with fine mesh pre-simulation. It is noted that using fewer elements to perform this pre-simulation does not impact noticeably on stiffness estimation over the three runs: R1, R2 and R3. Figure 9 also shows the MPE achieved without any pre-simulation. It can be seen that without the resetting of μ_n values at the second convergence the system tends to give greater error. For these simulations $N = 10,000$, $D = 49$ and injection is applied 5 times.

Figure 9: Mean percentage error without pre-simulation, with coarse mesh pre-simulation and with fine mesh pre-simulation

3.4 Number of Deflection Readings

The CE method estimates plate stiffness given deflection readings at D locations on the plate. Figure 10 shows where the readings are located on the plate, for $D = 49$, 16 and 9.

Figure 10: Location of deflection readings for different values of measurement points D

Several simulations were carried out on a damaged plate with different numbers of deflection readings. For these simulations damage is defined by ‘DEA’, $N = 10,000$ and injection is applied 5 times. As before, dark areas indicate that E_x has been overestimated by CE, while white areas indicate E_x has been underestimated by CE. Where only a grey bar is shown, CE has estimated E_x to within $\pm 3\%$ of the actual value. With $D = 49$, CE is operating with a deflection reading at every internal node and stiffness estimation is reasonably good (Figure 6). The damaged area is clearly identified and healthy elements are indicated elsewhere. When the number of readings is reduced to $D = 16$, the system still shows the lowest stiffness values in the correct location (Figure 11(a)). Even with $D = 9$ the algorithm indicates the lowest stiffness values in the correct elements (Figure 11(b)). However the algorithm underestimates E_x in elements surrounding the damage, thereby overestimating the area of the plate affected by damage. With $D = 9$, the stiffness in damaged elements is overestimated by up to 33%.

Figure 11: (a) Stiffness estimation from CE for $D = 16$, (b) Stiffness estimation from CE for $D = 9$

Figure 12 shows the output MSE for $D = 49$, $D = 16$ and $D = 9$. The MSE resulting from repeating the algorithm three times (R1, R2 and R3) for each D value is shown in the figure, to account for the variability in the system. As the number of readings is reduced, higher values of MSE are associated with the algorithm output,

Figure 12: Output MSE for various values of measurement points D

4 Stiffness Estimation in the Presence of Noise

Static load testing has the advantage of having fewer sources of noise than dynamic testing. In order to test the algorithm's sensitivity to noise, simulations carried out for damage event 'DEA' with $N = 10,000$, with 5 applications of injection and deflection readings at 49 nodes, were contaminated. For this purpose, Gaussian white noise is employed, with zero mean, and standard deviation (σ_{NOISE}) proportional to the mean of the clean signal (σ_{SIGNAL}), in this case the set of deflection readings measured from the plate under investigation. The quantity of noise is defined by the signal-to-noise ratio (SNR) given by Equation (4). Noise defined in this way is not proportional to the individual readings it is applied to, but rather the set of readings as a whole.

$$\sigma_{NOISE} = \frac{\sigma_{SIGNAL}}{SNR^2} \quad (4)$$

Figures 13 and 14 show the estimation of stiffness for each individual plate and the mean percentage error for all plates respectively and two levels of noise: relative errors in measurements of 5% (SNR=20) and 10% (SNR=10). The loss of accuracy in the predicted stiffness from using a clean signal (Figure 6(b)) to a SNR of 20 (Figure 13(a)) is relatively small. As noise increases, errors in stiffness get more pronounced near supports and the severity of the damage is not predicted as accurately. Even so, for SNR of 10, it is still possible to identify a clear area with smaller average stiffness where the damage is located (Figure 13(b)). The mean percentage error due to a SNR of 10 depends on the randomness of the process, but it can be more than twice the MPE of a clean signal (Figure 14).

Figure 13: (a) CE output for SNR = 20, (b) CE output for SNR = 10

Figure 14: Mean percentage error for simulations with various quantities of noise (SNR)

5 Conclusions

An algorithm based on the CE method has been developed to estimate the longitudinal stiffness of damaged plates. The system has been applied to different damage events and parameters of the algorithm have been varied to assess the sensitivity of the system. The CE method presented here has been shown to be capable of indicating the location of damage and estimating damage severity. The system is most effective with all deflection data available to it. Even as the number of data points available to the system is reduced, or the data is corrupted with noise, the algorithm still gives a reasonable indication of the damage location. A system of pre-simulation has been applied to bring the elemental means closer to their true values. This pre-simulation has been successfully carried out with fewer plate

elements than the plate mesh used in the final algorithm. This ability to operate with a coarser mesh may be useful in applying the CE method to more complex systems. Therefore, the proposed method could be adapted to detect irregularities in structural parameters other than longitudinal stiffness, such as transverse stiffness, or loss in member thickness.

References

- [1] E.P. Carden, P. Fanning, "Vibration based condition monitoring: a review", *Structural Health Monitoring*, 3, 355-377, 2004.
- [2] S.W. Doebling, C.R. Farrar, M.B. Prima, D.W. Shevitz, "Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: a literature review", Los Alamos National Laboratory Report LA-13070-MS, 1996.
- [3] P. Luo, D. Zhang, L. Wang, D. Jiang, "Structural Damage Detection Based on a Fiber Bragg Grating Sensing Array and a Back Propagation Neural Network: An Experimental Study", *Structural Health Monitoring*, 9, 5 - 11, 2010.
- [4] G. Buda, S. Caddemi, "Identification of concentrated damages in Euler-Bernoulli beams under static loads", *Journal of Engineering Mechanics*, 133, 942-956, 2007.
- [5] M. Sannayei, G. Imbaro, J. McClain, L.C. Brown, "Structural model updating using experimental static measurements", *Journal of Structural Engineering*, 123, 792-798, 1997.
- [6] J-H. Chou, J. Ghaboussi, "Genetic algorithm in structural damage detection", *Computers and Structures*, 79, 1335-1353, 2001.
- [7] F. Bakhtiari-Nejad, A. Rahai, A. Esfandiari, "A Structural Damage Detection Method Using Static Noisy Data." *Engineering Structures*, 27, 1784-1793, 2005.
- [8] B.J. Walsh, A. González, "Assessment of the Condition of a Beam Using a Static Loading Test", *Key Engineering Materials*, 413-414, 269-276, 2009.
- [9] R.Y. Rubinstein, D.P. Kroese, "The Cross-Entropy Method", Springer, New York, United States, 2004.
- [10] C. Rowley, "Moving Force Identification of Axle Forces on Bridges", PhD thesis, University College Dublin, 2007.
- [11] W.F. Carroll, "A Primer for Finite Elements in Elastic Structures", John Wiley & Sons Inc., New Jersey, United States, 1998.