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Concentration Curves, Inequality
and Tax Reform

by
David Madden

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Concentration Curves, Inequality and Tax Reform

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ABSTRACT: This paper applies the concept of welfare dominance using concentration curves to household data for Ireland. It identifies marginal tax reforms which would be welfare-enhancing for all social welfare functions satisfying relatively weak restrictions. It also examines cases where stronger restrictions need to be imposed on the social welfare function to yield welfare-dominance. These stronger restrictions which we label limited third degree stochastic dominance extends the range of welfare-enhancing marginal tax reforms.

Keywords: Concentration curve, Lorenz curve, stochastic dominance.

JEL Classification: H2, D6.

This is strictly a preliminary draft and not for quotation without the permission of the author. Comments and suggestions welcome.

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Concentration Curves, Inequality and Tax Reform

1. Introduction.

Applied optimal tax design and tax reform analysis typically involves the specification of a social welfare function. This raises the issue of the sensitivity of the optimally calculated tax rates and/or directions of reform to the particular social welfare function employed. In an attempt to overcome this problem, Yitzhaki and Thirsk (1990) and Yitzhaki and Slemrod (1991) introduce a method of identifying (indirect) tax reforms which would be approved by all individuals who can agree upon certain relatively weak assumptions with regard to the social welfare function. Thus, to a large degree, the sensitivity of the direction of welfare improving tax reforms to the exact properties of the social welfare function is avoided.

Central to this approach to tax reform analysis is the concept of a concentration curve, which is very similar to the well-known Lorenz curve from income distribution analysis. Indeed, the welfare analysis of income distributions using Lorenz curves and tax reforms using concentration curves is formally very similar. Atkinson (1970) and Shorrocks (1983) show how income distributions may be ranked using the concepts of Lorenz and generalised Lorenz curves. They show that, provided generalised Lorenz curves do not intersect, then income distributions can be ranked in accordance with social welfare functions that satisfy relatively weak restrictions. Similarly, provided concentration curves do not intersect then

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1 For an example of the former see Deaton (1977) and for the latter see Madden (1995a).

2 See Lambert (1993) for an excellent introduction to and summary of this literature.
it is possible to identify welfare-improving commodity tax changes which would be approved by a social welfare function satisfying relatively weak restrictions. If generalised Lorenz curves do intersect then it may still be possible to rank distributions but only if stronger restrictions are imposed on the social welfare function. This paper explores whether similar restrictions may be imposed on social welfare functions in order to identify welfare-improving commodity tax changes when concentration curves intersect.

The layout of this paper is as follows: Section 2 formally defines concentration curves and shows how they are used to identify welfare-improving tax reforms. We then apply this methodology to Irish household data and empirically investigate the scope for tax reform. Section 3 examines the case where concentration curves intersect. We outline the restrictions which are imposed upon social welfare functions to rank income distributions and explore whether restrictions can be applied to the case of concentration curve analysis, once again applying the methodology to Irish data. Section 4 provides concluding comments.

2. Concentration Curves and Tax Reform.

We assume that tax policies are evaluated according to an additively separable social welfare function. Formally

\[
W = \sum_{h} w[y_{h}^{*}(y_{h}, P_{1}, \ldots, P_{s})]
\]

where \(y\) is income, the \(P\)'s are prices, \(v\) is indirect utility and \(w\) is the social evaluation of the utility of individual (or household) \(h\). The social evaluation of the marginal utility of income, denoted \(\beta\), is a function of \(y\) (and prices) only. Thus

\[
\beta(y) = \frac{\partial w}{\partial v} \cdot \frac{\partial v}{\partial y}
\]

is a declining function of \(y\) with \(\partial \beta/\partial y < 0\).

Suppose the government is considering a marginal tax reform consisting of a small increase in the tax on commodity \(t\) and a small decrease in the tax on commodity \(s\), so that overall revenue is unchanged. Let \(x_{i}\) denote the consumption of good \(i\) by household \(h\), where households are arranged in nondecreasing order of income and let \(X_{i}\) denote total consumption of commodity \(i\). Let \(R\) be tax revenue and assuming there are \(K\) taxed commodities and that producer prices are normalised to 1, we have

\[
R = \sum_{i} x_{i} \tilde{x}_{i}
\]

where \(r\) denotes the tax (subsidy) rate. Reducing the tax on commodity \(s\) while increasing the tax on commodity \(t\) so as to keep revenue unchanged implies that

\[
dR = \partial P_{i} \times \sigma_{i} + \partial P_{s} \times \sigma_{s}
\]

where \(\sigma_{i}=1+\sum_{i}(r_{i}/X_{i})(\partial X_{i}/\partial r_{i})\). \(P_{i}\) refers to the consumer price \(i\), so that \(P_{i}=1+r_{i}\). The term \(\sigma_{i}\) reflects the revenue effect of a change in \(r_{i}\); in general it will depend upon all tax rates and the properties of the demand functions. (4) can be rewritten as

\[
\frac{\partial}{\partial r_{i}} = \frac{\partial P_{i} \times \sigma_{i}}{\partial r_{i}} = \frac{\partial P_{s} \times \sigma_{s}}{\partial r_{i}}
\]

Much of this section follows the exposition in Yitzhaki and Snirood (1991).

The results also apply to an increasing, S-concave social welfare function.

\[
3\text{ In this analysis as our "income" measure we will use expenditure per equivalent adult.}
\]
\[ dP_i = -\frac{X_i}{X} \alpha_s dP_s \]  

(5)

where \( \alpha_s = (\alpha_s/\alpha_t) \) can be regarded as the ratio of the marginal costs of public funds.\(^6\) If either \( \alpha_s \) or \( \alpha_t \) is negative so that \( \alpha_s < 0 \), then both prices can be decreased leaving revenue constant. We can refer to this unusual case as a commodity specific Laffer effect.\(^7\)

Now consider the change in the utility of household \( h \) following the above change in tax rates. Omitting the index \( h \) for simplicity of presentation this is given by

\[ dv = v_i dP_i + v_i dP_s = -v_i (x_i dP_i + x_i dP_s) \]  

(6)

where \( v_i \) is the derivative of \( v \) with respect to \( P_i \) and the second equality arises from Roy's identity. Substituting (5) in (6) we obtain

\[ dv = -v_i \left( \frac{x_i}{X_i} \alpha_s \right) x_i dP_s \]  

(7)

Since \( v_i \) and \( X_i \) are positive and \( dP_s \) is negative, household \( h \) will gain from the reform if the expression in square brackets in (7) is positive. If this expression is nonnegative for all \( h \), then the tax reform is a Pareto improving tax reform.\(^8\)

A necessary condition that all additive concave social-welfare functions would show that the above tax reform be welfare-enhancing is that the reform does not lower the welfare of the poorest household (since otherwise a Rawlsian social welfare function would show the reform to be welfare-lowering). Thus if household 1 is the poorest household this necessary condition implies that \( x_1/X_1 \alpha_s (x_1^1/x_1^1) \) be nonnegative. This necessary condition can be re-expressed by saying that the share of the expenditure of the poorest household on commodity \( t \) be greater than a constant times its share of expenditure of commodity \( t \).

This necessary condition can be extended to the poorest and next poorest household together. By assumption the marginal utility of income of the second household is lower than that of the first household and so another necessary condition is that the gain in real income for the first household be greater than the loss, if any, for the second. Thus analogous to the situation above we obtain a second necessary condition:

\[ \frac{(x_1^1 + x_2^1)}{X_1} - \alpha_s \frac{(x_1^1 + x_2^1)}{X_1} > 0 \]  

(8)

The set of necessary conditions that apply to each group of households from 1 to \( H \) is thus

\[ \frac{\sum_{k=1}^{k-1} x_k^1}{X_1} - \alpha_s \frac{\sum_{k=1}^{k-1} x_k^1}{X_1} \geq 0 \]  

(9)

for \( k = 1, \ldots, H \). (9) can also be expressed in its continuous version:

\[ \int_{0}^{\infty} \left( \frac{x(t)}{X} - \alpha_s \frac{x(t)}{X} \right) f(t) dt \geq 0 \]  

(9a)

for all \( y \), where \( f(t) \) is the density function. In other words, given the current constellation of tax rates, a revenue-neutral marginal change in taxes, consisting of a reduction in the tax on good \( s \) accompanied by an increase in the tax on good \( t \) will lead to an increase in welfare for all concave social welfare functions. Intuitively, the distribution of the burden of the tax on good \( s \) is more skewed towards the less well-off and so a marginal reduction in that burden accompanied by a marginal increase in a tax whose burden falls relatively more on

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6. \( \alpha_s \) is also equal to \( \rho_s/\rho_t \), the ratio of marginal revenue costs as outlined in Madden (1995), (or alternatively \( \lambda_s/\lambda_t \) using the notation of Ahmad and Stern (1984), providing we ignore equality considerations and assume that the social marginal utility of income is the same for all households.

7. While such effects may appear empirically unlikely they have been observed by the author in certain cases. See Madden (1995b).

8. See Guesnerie (1977) for the conditions for a Pareto-improving tax reform.
the better-off will lead to an overall rise in social welfare, while controlling for efficiency factors.

We can give an intuitive diagrammatic interpretation to (9) and (9a). Suppose we have on the horizontal axis households ordered according to their income, while on the vertical axis we have the cumulative percentage of the total expenditure on a specific commodity that is spent by households whose incomes are less than or equal to the specified income level. The resultant curve is called the concentration curve for the commodity in question.\(^5\) It is similar to a Lorenz curve in that it passes through the origin, but unlike the Lorenz curve it need not always be strictly increasing.\(^6\) In figure 1 we show concentration curves for a number of commodities using data from the Irish Household Budget Survey of 1987.

The expression in (9) and (9a) is the difference between the height of the concentration curve of commodity \(s\) and the height of the concentration curve of commodity \(t\) multiplied by a constant (where this constant takes account of the efficiency implications of the reform). Thus for all additive concave social welfare functions to show that our hypothetical tax reform increases welfare it is necessary that the concentration curve of commodity \(s\) with respect to income is at least as high as the concentration curve of commodity \(t\) multiplied by a constant at each point of the income distribution. The expressions in (9) and (9a) can thus be regarded as difference in concentration curves, which we label \(\text{DCC}_s(t)\). The curve \(\text{DCC}_s(t)\) has the following properties: (a) \(\text{DCC}(0)=0\), (b) \(\text{DCC}(y')=1-\alpha_s\), (c) \(\text{DCC}(s)=x_s(Y)/X_s\) i.e. its critical values correspond to the crossing points of \(x_s(Y)/X_s\) and \(x_t(Y)/X_t\), and where \(y^\prime\) is the highest observed income. What Stemrod and Yitzhaki essentially show is that for all concave social welfare functions, the condition \(\text{DCC}_s(t)\geq 0\) is sufficient for what we can term welfare dominance i.e. a small tax decrease in the tax on \(s\) accompanied by a small increase in the tax on \(t\) (leaving revenue unchanged) increases social welfare.\(^1\) If and only if these "shifted" (i.e. taking account of the constant \(\alpha_s\)) concentration curves do not intersect will all additive social-welfare functions show that the tax change increases welfare.

Yitzhaki and Stemrod (1991) refer to these rules as "marginal conditional stochastic dominance" rules and they have been applied to tax and expenditure data from Israel and the Co-te d'Ivoire (Yitzhaki and Thursk (1990), Yitzhaki and Stemrod (1991)). However in neither of these cases did the authors take explicit account of efficiency conditions i.e. in both cases they assumed that \(\alpha_s=1\) for all pairwise comparisons. We have applied these rules to the case of Ireland using 1987 Household Budget Survey data and we have explicitly taken account of efficiency considerations by calculating \(\alpha_s\) for each pairwise comparison between commodities.\(^1\) As an example in figure 2 we show the concentration curve for food and the shifted concentration curve for services. Since the shifted concentration curve for services lies below that of food at the highest observed income, we can conclude that in this case \(\alpha_s<1\), and that we would recommend that food be subsidised and services taxed at the margin even with an almost linear social welfare function. In figure 3 we show the DCC curve for food and services. It is positive for all levels of income, including the

\(^5\) For a detailed discussion of concentration curves and their properties, see Yitzhaki and Oskin (1988).

\(^6\) Note however, that the concentration curve for a direct proportional tax on income will be the Lorenz curve.

\(^\text{Note the similarity with the marginal indirect tax reform literature of Ahmad and Stern (1984) and Madden (1995a). These papers however explicitly specify the social welfare function, whereas here we merely restrict the social welfare function to be increasing and S-concave.}\)

\(^\text{The demand responses used to calculate the } \alpha_s \text{ come from Madden (1995b).}\)
highest, indicating that food welfare dominates services for all concave social welfare functions. In figure 4 we show the DCC curve for food and alcohol. This curve is positive for the first seven deciles but negative for the last three. Thus we cannot conclude that food welfare dominates alcohol since it would be possible to construct a social welfare function with only a limited degree of concavity which would show that welfare falls following the imposition of a small tax on alcohol and a small subsidy on food.

Table 1 shows the percentage of pairwise comparisons yielding unambiguous cases of welfare dominance. We calculated concentration curves for ten commodities using the Irish Household Budget Survey of 1987. The ten goods were: food, alcohol, tobacco, clothing and footwear, fuel and power, petrol, transport and equipment, durables, other goods and services. Given n goods there are n(n-1)/2 possible pairwise comparisons and thus we have 45 possible comparisons of concentration curves. The second column shows these pairwise comparisons which yield dominance (what we refer to as second-degree dominance). An asterisk indicates that dominance works "the other way". Thus in 32 cases i.e. about 71%, we can unambiguously identify welfare dominance, taking as given our demand response parameter \( \alpha_n \). The high percentage of cases yielding dominance indicates that there is considerable scope for tax reform in the Irish indirect tax system, for a wide range of possible social welfare functions.13

Before discussing those cases where we do not have unambiguous welfare dominance, we will discuss briefly the role of the demand response parameter \( \alpha_n \). Obviously the value of \( \alpha_n \) has a critical effect on whether or not welfare dominance is observed. The calculated value for \( \alpha_n \) will come from parameters estimated from empirical demand studies and thus

\[ \alpha_n \] will have an implied standard error. Were such standard errors available then it would be possible to construct confidence intervals around DCC curves, which would at least take explicit account of the uncertainty surrounding the value of these parameters.14 We do not present standard errors for our calculated \( \alpha_n \) here, but nevertheless we believe it is preferable to explicitly include an estimated value rather than merely assume that all \( \alpha_n \) take on a value of unity.

As we can see from Table 1, only 71% of pairwise comparisons yield unambiguous welfare dominance. While this is quite a high percentage and shows considerable scope for tax reform, what can we say about the remaining 29%? As Yitzhaki and Stremrod (1991) point out there are at least two directions in which this research can be pursued. Firstly, we could examine tax reform "packages" involving more than just pairwise comparisons. This approach is adopted by Mayshar and Yitzhaki (1993). Alternatively, we could examine what further restrictions could be imposed on the social welfare function which might allow us to identify some form of welfare-dominance for the remaining 29% of pairwise comparisons. It is this second approach which we concentrate upon in section 3.

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13 This finding essentially replicates the findings from Madden (1995a) where a specific welfare function was used and the estimated marginal social cost parameters showed a wide divergence (see footnote 6).

14 See Decoster and Schokkaert (1990) for an example of calculation of standard errors for marginal tax reform parameters. These authors and Madden (1993) both find that the ranking of goods according to marginal tax reform parameters is more sensitive to demand responses than to the assumed degree of inequality aversion.
3. Third-Degree Stochastic Dominance.

The rules for welfare dominance which we have outlined above arise from rules of second-degree stochastic dominance which can be applied to any concave social welfare function. One approach to identifying restrictions which may be imposed upon the social welfare function is to examine rules of third-degree stochastic dominance. These rules can be applied to social welfare functions which are not only concave but also have the property that the third derivative of welfare with respect to income be positive. On an intuitive basis this implies that our welfare functions obey not just Dalton’s Principle of Transfers but what Kolm terms the Principle of Diminishing Transfers (Kolm, 1976). In other words, not only should every transfer from rich to poor increase welfare but transfers between persons with a given income difference are valued more when the income is lower rather than higher. In the limit, this implies that the small transfer most valued is that between a very poor person, which not everyone might find intuitively appealing.

A general definition of third-degree stochastic dominance is as follows: if \( F(x) \) and \( G(x) \) are cumulative probability distributions, where \( x \) is a random variable representing the outcome of a prospect, then the prospect \( F(x) \) is not preferred to the prospect \( G(x) \) for all utility functions satisfying \( U'(x) > 0, U''(x) \leq 0 \) and \( U'''(x) \geq 0 \), if and only if

\[
\int_a^b \int_a^b (F(x) - G(z)) \, dz \, dx \geq 0, \quad \forall \, x \in [a,b]
\]

(10a)

and

\[
\int_a^b (F(y) - G(y)) \, dy \geq 0
\]

(10b)

This condition can be translated into a condition on the DCC curve. As Yitzhaki and Slemrod point out (1991, p. 486, footnote 13) a necessary and sufficient condition for third-degree marginal conditional stochastic dominance is that the area between the DCC curve and the horizontal axis (i.e. the integral of the DCC curve) be positive everywhere and that \( DCC(1) \geq 0 \). Thus we require

\[
\int_0^1 DCC_\alpha (t) \, dt > 0, \quad DCC_\alpha (1) \geq 0
\]

(11)

Thus the integral of the DCC curve must be always positive and the DCC curve at the highest point on the cumulative distribution of income must be positive. If this condition holds, then commodity \( s \) will be welfare dominant over commodity \( t \) for all concave social welfare functions which also satisfy the condition \( W''(y) \geq 0 \). Effectively, this provides an ordering between two commodities whose concentration curves intersect, with commodity \( s \) above commodity \( t \) in the neighbourhood of the lowest income, and where commodity \( s \) dominates commodity \( t \) in "efficiency" terms, i.e. \( \alpha_s \leq 1 \). An example of such an outcome is drawn in figure 5.\(^{14}\)

Intuitively, however, we are unlikely to observe such commodities, since it would imply that commodities would "reverse rankings" twice in terms of their income elasticities and this is borne out in Table 1 where we find no pairwise comparisons satisfying such conditions which we label "3rd degree". Thus to observe 3rd degree stochastic dominance, we would require that over an initial range of income the proportion of the total expenditure of good \( s \) accounted for by households is greater than that of good \( t \), over a second range of income this ranking is reversed, and then over the final range the ranking is either reversed.

\(^{14}\) For a discussion of third degree stochastic dominance in income distribution analysis, see Shorrocks and Foster (1987).
again, or at least the (shifted) concentration curves are equal at the highest recorded income.

What about the case where the integral of the DCC curve is everywhere positive but \( \text{DCC}(y') \leq 0 \)? Are there any further restrictions we can place upon the social welfare function which would permit an ordering in this case? To answer this we turn to the parallel literature on ranking income distributions by means of Lorenz and generalised Lorenz curves.

The normative comparison of income distributions via Lorenz curves dates back to the contribution by Atkinson (1970). His theorem was generalised by Shorrocks (1983) who defined what he termed a generalised Lorenz curve i.e. a Lorenz curve scaled upwards by the mean value of the distribution. Shorrocks showed how welfare from distribution \( y \) is superior to welfare from distribution \( y' \) if and only if the generalised Lorenz curve for distribution \( y \) is always above that of distribution \( y' \).

Suppose \( L(y, p, \mu) \in [0, 1] \) is the Lorenz curve for distribution \( y \), then the generalised Lorenz curve for \( y \) is \( GL(y, p, \mu) = \mu L(y, p) \). Thus if \( W(y) \) is welfare, Shorrocks' theorem 2 states (Shorrocks, 1983, p.6):

\[
W(y) = W(y') \text{ for all } W(y) \in W_1 \text{ iff } GL(y, p, \mu) \geq GL(y', p) \forall p
\]  
(12)

where \( W_1 \) is the set of all non-decreasing S-concave welfare functions.

Recalling the definition of a generalised Lorenz curve, this implies

\[
\int y \cdot f(y) \, dy - \int y \cdot g(y') \, dy \geq 0
\]  
(13)

where \( f(y) \) is the probability density function for \( y \) and \( g(y') \) is the probability density function for \( y' \). Integrating the above expression by parts we obtain

\[
\int [G(y) - F(y)] \, dy \geq 0
\]  
(12)

which is the condition for second order stochastic dominance of distribution \( y \) over distribution \( y' \).

Shorrocks' theorem however does not enable us to rank distributions whose generalised Lorenz curves intersect. Where generalised Lorenz curves intersect once and distribution \( y \) Lorenz dominates distribution \( y' \) at the lowest observed income level and \( \mu_y \geq \mu_y' \), then \( y \) will be ranked above \( y' \) by all welfare functions satisfying \( W^n \geq 0 \) i.e. the condition for third-degree stochastic dominance referred to above. Thus where concentration curves and generalised Lorenz curves intersect further identification of welfare dominance by third-degree stochastic dominance is possible.

However, we have seen that when examining DCC curves we are unlikely to observe conditions of third-degree stochastic dominance. Are there any further restrictions which might be imposed upon the welfare function to enable us to identify welfare dominance? It turns out that in the income distribution literature, ranking of intersecting generalised Lorenz curves is possible providing we are prepared to place certain limits on the degree of concavity of the social welfare function.

Thus in terms of the income distribution literature the case we are examining is where the generalised Lorenz curves of distributions \( y \) and \( y' \) cross once \( y \) Lorenz dominates \( y' \) in the neighbourhood of the lowest observed income level and \( \mu_y \leq \mu_y' \). Presuming we restrict the welfare function to those satisfying \( W' > 0 \), \( W^> < 0 \) and \( W^n > 0 \), then Lambert proves the following theorem (Lambert (1993, pp. 76-77)):

Suppose we have two income distributions, \( y \) and \( y' \) whose cumulative density functions are respectively given by \( F(y) \) and \( G(y') \). \( \mu_y < \mu_y' \). \( F > \mu G \) (i.e. distribution \( y \) is preferred to distribution \( y' \) by the Rawlsian leximin criterion) and \( GL_y \) and \( GL_{y'} \) cross once. If \( \sigma_y < (\mu_y - \mu_y')(2\mu_y - \mu_y') \), then \( \int W(y) f(y) dy \geq \int W(y') g(y') dy \) such that:
\[ q_w(y) \geq \frac{y'(\mu_y - \mu_y)}{[(e^y - e^{-y}) - (\mu_y - \mu_y)(2y - \mu_y)]} > 0 \] (15)

where \( q_w(y) = -yW'(y)/W(y) \) is the negative of the elasticity of the social marginal utility of income and \( y' \) is the highest observed income.\(^{17}\)

If we regard \( q_w \) as a measure of inequality aversion, then intuitively the above result puts a lower bound on the degree of inequality aversion which permits unanimous preference for \( y \) over \( y' \) by social welfare functions satisfying the principle of diminishing transfers.

This theorem of Lambert's begs the question of whether an analogous result can be derived for situations where concentration curves cross. Thus the cases we are interested in are those where \( \int DCC(y) \geq 0 \), but \( DCC_a(y') < 0 \). We label such cases in Table 1 under the heading "Limited", since ranking is possible only for a limited degree of concavity of the social welfare function. As can be seen from Table 1, a further five or 11% of possible pairwise comparisons can be ranked using this criterion. As an example in figure 6 we show the case of food and petrol. The DCC curve cuts the horizontal axis at about the ninth decile i.e. income levels in excess of approximately £505, but the integral of the DCC curve is positive for all levels of income. Thus food welfare dominates petrol for all social welfare functions satisfying the principle of diminishing transfers and for a limited degree of concavity (for this case the calculated value for \( q_w \) was around 2.5).

4. Conclusion.

In this paper we have applied Yitzhaki and Stemrod's concept of welfare dominance in the context of shifted concentration curves to Irish data. Not only have we identified cases of welfare dominance but we have also empirically examined what proportion of pairwise comparisons between shifted concentration curves are likely to yield unambiguous welfare dominance. We discover that quite a high proportion, 71%, will yield unambiguous welfare dominance and, borrowing from the income distribution literature, we suggest an alternative procedure for ranking the remainder. We find that this condition, which we label "limited 3rd degree stochastic dominance" can provide rankings for a further 11% of pairwise comparisons. There is still a remaining 18% of pairwise comparisons which cannot be unambiguously ranked.

\(^{17}\) Note that the condition for the denominator in (15) to be positive is equivalent to the condition that \( \int_s^y S(y) > 0 \) and \( S(y') < 0 \), where \( S(x) = \int_s^x [G(y) - F(y)] dy \) i.e. the cumulative area between the two distribution functions. See Lambert (1993, pp. 63-77).
REFERENCES


### Table 1: Dominance Results for Pairwise Comparisons

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### Table 1 (con.): Dominance Results for Pairwise Comparisons

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* indicates that dominance works in reverse direction e.g. tobacco dominates food.