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An Equilibrium Search Model of the Informal Sector

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Abstract

We use an equilibrium search framework to model a formal-informal sector labour market where the informal sector arises endogenously. In our model large firms will be in the formal sector and pay a wage premium, while small firms are characterised by low wages and tend to be in the informal sector. Using data from the South African labour force survey we illustrate that the data is consistent with these predictions.

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Section I: Introduction

One of the main differences between labour markets in developing compared to developed economies is the existence of large informal sectors. For example, in Africa the informal sector is estimated to absorb about 60 per cent of the urban labour force.[2] Importantly in this regard, it is generally assumed, and empirically substantiated by much of the literature[3], that workers in the informal sector are paid less than their formal sector counterparts. However, theoretically it is not clear why this should be the case. While a tax wedge would explain differences in gross wages, if workers can move between sectors then net wages should surely be equalised. Earlier papers in the literature such as Lewis (1954) or Harris and Todaro (1970) assumed a dual labour market structure where workers earned rents in the primary sector and secondary sector workers queued for good jobs. There are of course many models that could be used to justify why workers in particular sectors would earn wage premiums – as, for example, efficiency wage[4] and union models – but applying these to explain a wage premium for formal sector employees would mean arbitrarily assuming that formal sector workers earn rents because of some exogenously imposed feature that for some reason is more relevant to the formal rather than the informal sector.

In this paper we use an equilibrium search framework, which is a modified version of Burdett and Mortensen (1998), to model the formal/informal sector labour market where the informal sector emerges endogenously.[5] More specifically, firms

[2] See http://www.ilo.org/public/english/employment/skills/informal/who.htm

[3] See, for example, Mazumdar (1981), Heckman and Hotz (1986), Pradhan and Van Soest (1995), Tansel (1999), and Gong and Van Soest (2002).

[4] Jones (1983) uses the shirking efficiency wage model to characterise the formal sector in a model with minimum wages

[5] Burdett and Mortensen (1998) outline the equilibrium search framework that has become increasingly popular and can be seen as providing a basis for modern monopsony models of the labour market [see Manning (2003)].

post wages and workers may work in the formal sector or may opt for a tax free outside option, which could be viewed as informal sector self-employment, as discussed and modelled by Albrecht et al. (2005). We find that in this set-up formal sector employees do indeed earn rents relative to their informal counterparts in the model. However, this is not because they are formal sector employees, but because in our model large firms will pay higher wages and have the incentive to stay in the formal sector. Intuitively it arguably makes sense that small firms would be the most difficult for the government to find and the most likely to stay in the informal sector. Indeed, a number of theoretical models [Fortin et al. (1997) and Rauch (1991), for example] impose this assumption. Moreover, many empirical studies seem to confirm that informal sector workers are concentrated in small firms.[6] As a matter of fact, small enterprise size is part of the ILO definition of the informal sector and has been used in a number of papers as a proxy for such.

A search model where it is difficult for workers and firms to find each other seems like a natural way to model the labour market with an informal sector in developing countries, where it is often argued that there are no clear channels for the exchange of labour market information.[7] There are other papers in the literature that have used a search-matching framework to model the informal labour market. For example, Albrecht *et al* (2005) extend the Mortensen and Pissarides (1994) matching model to incorporate a self-employed informal sector where there is heterogeneity in workers' productivity in that more productive workers may opt to wait for a formal sector job, while others may select into the informal sector. Also, Boeri and Garibaldi (2005) develop a matching model with supervision where workers in the informal sector cannot avail of unemployment benefit, and show that matches found not paying

[6] See, for instance, Tybout (2000).
[7] See, for example, Hussmanns (1994) or Byrne and Strobl (2004) .

tax are dissolved. Their model suggests that policies aimed at reducing the size of the shadow economy may increase unemployment. Alternatively, Fugazza and Jacques (2001) incorporate psychic costs as part of the costs of being in the informal economy in a matching model where workers direct their search at informal sector firms. However, it is important to emphasize that while the papers using the matching framework just noted focus on exogenously given worker heterogeneity. In the equilibrium search framework we adopt in this paper the informal sector emerges endogenously without arbitrarily imposing any differences in the two sectors other than that larger firms are more likely to be caught defaulting on their tax.[8]

A key prediction of our equilibrium search framework is that large firms pay more even when there is no heterogeneity amongst either workers or firms ex ante. It is only in the case where there are no search frictions that the labour market is competitive and the formal/large firm size premium disappears. There is already some evidence that suggests that firm size may be a driving factor behind the often observed formal sector wage premium. For example, Pratap and Quintin (2005) find, using Argentinean data and semi-parametric techniques to deal with the selectivity issue inherent in estimating the possibility of a formal sector wage premium, that there is no difference in gross wages between informal workers and their formal sector counterparts and that the employer's size is crucial in making the wage premium 'disappear'[9]. Using a similar econometric techniques and rich South African data that allows a relatively precise measure of informal employment we confirm that firm size

[8] In our paper we interpret informality to mean tax avoidance rather than just any illegal activity. Schneider and Enste (2000) provide a survey of the general literature on shadow economies and its various definitions.

[9] Amaral and Quintin (2006) outline a theoretical framework where the only difference between informal and formal sector firms is that informal sector firms are seen as more likely to default on loans, have difficulty accessing credit and because of this, rely on self financing. Because of the complementarity between skill and capital, high skill capital intensive firms enter the formal sector and hire high skill workers. Thus in contrast to our model labour markets are competitive and wage differentials can be explained by differences in ability.

can explain away the formal sector wage premium, but only if one assumes, as appears reasonable, that informal sector workers do not pay taxes, as is assumed in our model.

One should note that while our equilibrium search model generates predictions that are in line with the empirical evidence - small low wage informal firms and large high wage formal firms - the model admittedly imposes a lot of structure. An obvious drawback is that if the structure we impose is incorrect one must worry that it may be driving the results. An advantage of this framework is, however, that we have a model that allows us to do comparative static analysis on the policy parameters and predict the long run change in the equilibrium wage distribution accounting for firm entry and exit. Given the amount of structure on the model though, it seems more reasonable to interpret the comparative static results as plausible examples rather than general results. Some of the comparative static results are surprising. In particular, we find that in the long run when we account for the impact of firm exit on the shape of the distribution, an increase in the tax rate may reduce the share of the informal sector for plausible parameter values. An increase in the enforcement/punishment parameter tends to reduce the share of the informal sector as one would expect.

The remainder of the paper is organized as follows. In the next section we present our model. In Section II we describe our data. Empirical evidence in support of results derived from our model are shown in Section III. Concluding remarks are given in the final section.

Section II: The Model

II.1 The Basic Set-Up

There is a mass of M identical employers and a mass L of identical workers in the economy. We normalise L to unity. The non-employment outside option is b and

employed workers have fixed productivity p.[10] There is random matching so that workers receive l offers at each instant and any offer is equally likely to come from any firm irrespective of the firm's size[11]. l is a Poisson arrival rate. The distribution of wage offers which we will solve for is *F(w).* Burdett and Mortensen (1998) derive the labour supply curve for individual firms. When we assume the arrival rate of job offers is same for employed and unemployed workers this is[12]:

$$n(w,F) = \frac{d\,l}{M\{d + l[1-F(w)]\}^2} \tag{1}$$

Using this expression for labour supply the profit of a firm is:

$$p(w,F) = n(w,F)(p-w) = \frac{d\,l(p-w)}{M\{d + l[1-F(w)]\}^2} \tag{2}$$

Given that employed and unemployed workers have the same arrival rate of job offers the reservation wage is just the benefit level b. The employment levels of firms paying the reservation wage and the highest wage $\overline{w}$ are:

$$n(b,F) = \frac{d\,l}{M[d + l]^2} \quad \text{and} \quad n(\overline{w},F) = \frac{l}{Md} \tag{3}$$

There is free entry and firms continue to enter until the expected flow of future profits equals entry costs. This implies that all firms make equal profits. In particular if fixed entry costs are k, we equate profits of reservation wage firms with firms paying any other wage $p(w,F) = (p-w)n(w,F) = p(b,F) = (p-b)n(b,F) = dk$ and solve for the wage distribution $F(w) = \frac{d+l}{l} - \sqrt{\frac{(p-w)}{klM}}$ (4)

[10] Traditionally this outside option b is viewed as unemployment benefits. In the context of developing countries it is perhaps more appropriately seen as self-employment or support for the non-employed by their family which is a relatively common feature of the developing world.

[11] See Manning (2003) pp284-286 for a discussion on the matching technology.

[12] We note here that the labour supply curve in Burdett and Mortensen allows for different arrival rates for unemployed ($?_0$) and employed ($?_1$) workers. The labour supply curve in this case, not normalising the mass of workers L to unity is: $n(w,F) = \frac{dl_1}{M\{d + l_1[1-F(w)]\}^2}[\frac{l_0}{l_1}(\frac{d+l_1}{d+l_0})L]$. That is it is just the labour supply curve in (2) (with ? replaced by $?_1$) times a constant.

This is the wage distribution as given in Burdett and Mortensen (1998) :[13]

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II.2 A Tax on Wage Income.

Here we modify the Burdett and Mortensen model by introducing a tax rate *t* on wage income that is paid by firms. Labour supply is still given by (1) once we solve for the wage distribution. There will be a Poisson arrival rate of tax inspectors, which is increasing in the size of the firm: $zn\ (w)^{b}$. We specify the penalty for defaulting as *x* times the firms per period tax bill *wtn(w).* To save on notation we define $s=xz$ as the parameter that determines the level of enforcement/punishment for defaulters and $\sigma=\beta+1$. The flow values of defaulting (d) and complying (c) firms in a stationary equilibrium are:

$$\begin{aligned} rV^{d} &= p^{d} - dV^{d} - swtn(w)^{s} \\ rV^{c} &= p^{c} - dV^{c} \end{aligned} \tag{5}$$

The flow value of the firm where *r* is the discount rate is the dividend stream (flow of profits) plus any capital gain/loss terms. The flow of profits for defaulting firms is $p^{d} = (p-w)n(w)$ and for complying firms: $p^{c} = [p - w(1+t)]n(w)$. Defaulting firms have a higher flow of profits than compliers at a given wage but, in addition to the exogenous arrival rate of negative shocks that close the firm d defaulting firms receive an expected flow of punishment $swtn(w)^{s}$ at each point in time. We note that the two policy instruments the government has are the tax rate *t* and the degree of punishment/enforcement s. Burdett and Mortensen (1998) assume *r=0* in their

[13] If we use the expression for k given above we get the formulation given in Burdett and Mortensen (1998): $F(w) = \frac{d+l}{l}[1 - (\frac{p-w}{p-b})^{\frac{1}{2}}]$.

derivation of the labour supply curve (1) and we also make this assumption. From (5) the value of defaulting and compliant firms respectively are:

$$V^d = \frac{(p-w)n(w) - swtn(w)^s}{d} \tag{6}$$

$$V^c = \frac{[p - w(1+t)]n(w)}{d} \tag{7}$$

Free entry ensures that these hold in equilibrium. Using (3) in (6) one can solve for this in terms of the reservation wage and get the equilibrium value of a firm in terms of the exogenous parameters. Comparing (6) and (7) it is straightforward to show that:

$$V^d > V^c \quad \text{if} \quad \frac{1}{s} > n^{s-1} \quad \text{and} \quad V^d < V^c \quad \text{if} \quad \frac{1}{s} < n^{s-1} \tag{8}$$

That is there is a critical level of employment n^{s-1} below which firms can always do better in the informal sector. We can use the expression for labour supply (1) in (8) to calculate the cut-off value of the wage offer distribution below which, firms will be defaulting[14]:

$$F^* = \frac{d+l}{l} - \sqrt{\frac{s^{\frac{1}{s-1}} d}{Ml}} \tag{9}$$

Free entry ensures that $V^d = V^c = k$. Imposing this free entry condition using (6) and (7) for the value of firms and (1) for labour supply we can calculate the relationship between the wage and offer distribution for defaulting and compliant firms:

$$w^d = \{p - \frac{kM}{l}[d + l(1-F)]^2\}\{\frac{M^{s-1}[d + l(1-F)]^{2(s-1)}}{M^{s-1}[d + l(1-F)]^{2(s-1)} + st(ld)^{s-1}} \tag{10}$$

[14] It is worth noting from (6) that even with a general production function y=y(n) ,where y is out put, equation (9) and (10) will hold.

$$w^c = \frac{p}{1+t} - \frac{kM}{l}\frac{[d + l(1-F)]^2}{1+t} \tag{11}$$

The wage in the lowest wage firm is b and since all other firms pay higher wages the value of the wage offer distribution will be zero at a wage b. Using $w=b$ and $F=0$ in (6) and setting the value of the lowest wage firm equal to entry costs k we can solve for the relationship between entry costs and the mass of firms in terms of the exogenous parameters:

$$k = \frac{(p-b)n(b) - bstn^s(b)}{d} = \frac{(p-b)l}{M(d+l)^2} - \frac{stbd^{s-1}l^s}{M^s(d+l)^{2s}} \tag{12}$$

In Figure 1 we graphically depict the inverse wage offer distribution of our model for two different tax rates, 10% and 30%, using (10) for values of F between zero and F* and (11) for values of F between F* and unity under assumed values for the exogenous parameters. The graph illustrates a wage offer distribution which is consistent with the stylised facts. Small low wage firms are in the informal sector and large high wage firms in the formal sector. While we will do some comparative static analysis later where both arrival rates of job offers (λ) and entry costs (k) are dependent on the mass of firms in equilibrium, Figure 1 plots the response to a tax change under the simpler assumption that these parameters are fixed when the mass of firms changes in accordance with (12). The wage distribution becomes more compressed in response to the higher tax rate as we would expect. Firms paying high wages must adjust there wage downwards in response to the tax, while the lowest wage firms are already paying the reservation wage and cannot lower the wage any further.

II.3 Endogenous Productivity

The basic Burdett and Mortensen model with homogeneous productivity across firms predicts a wage distribution with a lot of weight on the upper tail of the distribution whereas empirically it has been observed that the wage distribution generally has a long right hand tail. Mortensen (2003) discusses this issue and outlines a number of generalisations to the basic Burdett and Mortensen model where productivity varies across firms. These generalisations generate wage distributions that are more in keeping with empirically observed wage distributions. This can be where there is exogenous variation in firms' productivity and firms can choose the number of contacts with workers, or, alternatively, where firms may be allowed to invest in costly match specific or general capital, which generates differences in productivity. We will take the case where firms invest in match specific capital and apply our model of the informal sector to this set-up.

Within this framework we look at the model analysed earlier where the risk of detection for defaulters rises with firm size so that small low wage firms are in the informal sector. We will set up the profit function in general terms before distinguishing between the defaulting and compliant sectors. We assume that $j \in [d, c]$ so that $w_j = w$ when $j = d$ and $w_j = w(1+t)$ when $j = c$.

Mortensen (2003) gives a detailed derivation of the labour supply curve in terms of the separation rate: $d^j(w) = \{d + l[1 - F^j(w)]$ of a firm offering wage w has at each point in time and the expected number of job offers accepted at each point in time for a firm offering wage w : $l h^j(w) = \dfrac{l d}{M\{d + l[1 - F^j(w)]\}}$. Using these definitions the labour supply curve (1) can also be written as:

$$n^j(w, F^j) = \frac{l h^j(w)}{d^j(w)} \tag{13}$$

We use this alternative notation for the labour supply curve because we wish to distinguish between the separation and offer acceptance rates. Firms invest in match specific human capital T which also costs the firm T. These sunk costs will be incurred every time an offer is accepted. Human capital enhances the productivity of a match according to the concave function *p(T)*, but the productivity gain of the investment is lost as soon as the worker leaves this firm. The cost of the investment T is multiplied by the number of matches but is unaffected by the separation rate. The profit function (2) in this case is:

$$p^{j}(w_j, F^{j}) = l h^{j}(w)[\frac{p(T) - w_j}{d^{j}(w)} - T] = \frac{l d}{M\{d + l[1 - F^{j}(w)]\}}[\frac{p(T) - w_j}{\{d + l[1 - F^{j}(w)]\}} - T] \tag{14}$$

We assume that $p(T)=pT^{a}$ and from the first order condition for the optimal choice of training T:

$$T = (pa)^{\frac{1}{1-a}}\{d + l[1 - F^{j}(w)]\}^{\frac{1}{a-1}} \tag{15}$$

Substituting (15) into the profit function one obtains:

$$p_{j}(w_j, F^{j}) = \frac{l d}{M}[(\frac{1-a}{a})(pa)^{\frac{1}{1-a}}\{d + l[1 - F^{j}(w)]\}^{\frac{2-a}{a-1}} - w_j\{d + l[1 - F^{j}(w)]\}^{-2}]$$

(16)

Equation (5) still gives us the value of each firm type and the amended versions of (6) and (7) for the value of defaulting and complying firms respectively can be written as:

$$V^{d} = \frac{l}{M}[(\frac{1-a}{a})(pa)^{\frac{1}{1-a}}\{d + l[1 - F^{d}(w)]\}^{\frac{2-a}{a-1}} - w\{d + l[1 - F^{j}(w)]\}^{-2}] - swt\frac{l^{s} d^{s-1}\{d + l[1 - F^{d}(w)]\}^{-2s}}{M^{s}} \tag{17}$$

$$V^{c} = \frac{l}{M}[(\frac{1-a}{a})(pa)^{\frac{1}{1-a}}\{d + l[1 - F^{c}(w)]\}^{\frac{2-a}{a-1}} - w(1+t)\{d + l[1 - F^{c}(w)]\}^{-2}] \tag{18}$$

One can see by comparing (17) and (18) that in equilibrium at a given wage and value of the distribution equation (8) still gives the condition that determines whether a firm can profit from moving to the defaulting from the compliant sector or vive-versa. Firms below the critical level of employment will default and firms above the critical level will comply. Given that (8) still holds equation (9) continues to give the fraction of wage offers in the defaulting sector. The equilibrium value of firms is given by looking at (17) for the lowest wage firm where F=0 and w=b.

$$k = \frac{l}{M}[(\frac{1-a}{a})(pa)^{\frac{1}{1-a}}(d+l)^{\frac{2-a}{a-1}} - b(d+l)^{-2}] - sbt\frac{l^s d^{s-1}(d+l)^{-2s}}{M^s} \qquad (19)$$

Next one can equate $V^c = V^d = k$ to solve for the equilibrium relationship between the wage and the wage distribution for both firm types:

$$w = \frac{M^{s-1}(\frac{1-a}{a})(pa)^{\frac{1}{1-a}}[d+l(1-F^d)]^{\frac{a+2(s-1)(a-1)}{a-1}} - kl^{-1}M^s[d+l(1-F^d)]^{2s}}{M^{s-1}[d+l(1-F^d)]^{2(s-1)} + st(ld)^{s-1}} \qquad (20)$$

$$w(1+t) = [\frac{1-a}{a}](pa)^{\frac{1}{1-a}}[d+l(1-F^c)]^{\frac{a}{a-1}} - \frac{kM}{l}[d+l(1-F^c)]^2 \qquad (21)$$

One can also solve for the highest wage by setting F=1 in (21).

We plot the distribution for the same assumed parameter values as in Figure 1 in Figure 2 at two different tax rates. Once again the graph illustrates a wage distribution that is consistent with the stylised facts. Low wage small informal firms, and large, high wage formal firms. In this case the inverse wage offer distribution is convex, indicating a small amount of weight in the upper tails, which is more in keeping with the empirically observed wage distributions. The higher tax rate compresses the wage distribution as in Figure 1.

II.4 Comparative Statics

Next we investigate the effect of changes in the policy variables on the percentage of workers who will be in the informal sector. Casual inspection of the cut-off value of the wage offer distribution where firms begin complying [equation (9)] suggests that the results for a change in the tax rate in particular will not follow our intuition. In particular the tax rate does not directly enter equation (9) and the only way a change in the tax rate affects the cut-off percentile where firms begin complying is through firm exit/entry. More precisely, it is apparent from (9) that if an increase in the tax rate or the punishment/enforcement parameters leads to firm exit where lambda is fixed then the fraction of offers from the formal sector will increase. One can also see from the formal analysis below that firm exit will occur and, even when lambda is variable, that these results will hold for a wide range of parameter values.

While the previous section derives the wage offer distribution, one generally observes the wage distribution in the data, i.e., the fraction of workers paid different wages or the fraction of workers in the informal sector etc. We note though that the wage distribution is a monotonic transformation of the wage offer distribution. In particular Mortensen (2003) shows that the wage distribution *G(w)* can be written as: $G(w) = \dfrac{dF(w)}{d + l[1 - F(w)]}$. For $z \in (s,t)$ one sees that:

$$\frac{dG^*}{dz} = \frac{d\dfrac{dF^*}{dz}[d + l(1-F^*)] + dF^*[-\dfrac{dl}{dM}\dfrac{dM}{dz}(1-F^*) + \dfrac{dF^*}{dz}l]}{[d + l(1-F^*)]^2} \qquad (22)$$

From (22) we see that when λ is fixed the sign of the derivative of the wage distribution is the same as the sign of derivative of the wage offer distribution. Moreover, if λ is increasing in the mass of firms $\dfrac{dM}{dz} < 0$ is a sufficient condition for

$\operatorname{sgn}\frac{dG^*}{dz} = \operatorname{sgn}\frac{dF^*}{dz}$ so that the comparative static results given for the wage offer distribution below will also apply to the wage distribution when $\frac{dM}{dz} < 0$. Next we define two conditions where $\varepsilon_{\lambda M}$ is the elasticity of λ with respect to the mass of firms:

Condition One: $p > b(1 + st)$

Condition Two: $\frac{l}{d} > \frac{e_{lM} - 1}{1 + e_{lM}}$

Appendix one shows that for the exogenous productivity case if lambda is constant, condition one is sufficient for $\frac{dM}{dz} < 0$ and if lambda is increasing in the mass of firms, Conditions One and Two are sufficient for $\frac{dM}{dz} < 0$.

We note that for both the exogenous and endogenous productivity examples in the two earlier sections, equation (9) gives us the cut-off value where wage offers are from the formal sector. Initially we will take the case where lambda is fixed. In this case the derivatives of F* with respect to the policy tax rate and punishment/enforcement parameter s are as follows:

$$\frac{dF^*}{dt} = \left.\frac{\partial F^*}{\partial M}\right|_l \frac{dM}{dt} = \frac{1}{2M}\sqrt{\frac{s^{\frac{1}{s-1}}d}{lM}}\frac{dM}{dt} \tag{23}$$

$$\frac{dF^*}{ds} = \frac{\partial F^*}{\partial s} + \left.\frac{\partial F^*}{\partial M}\right|_l \frac{dM}{ds} = \frac{1}{2}\sqrt{\frac{s^{\frac{1}{s-1}}d}{lM}}\left[\frac{1}{M}\frac{dM}{ds} - \frac{1}{s(s-1)}\right] \tag{24}$$

From (23) the sign of $\operatorname{sgn}\frac{dF^*}{dt} = \operatorname{sgn}\frac{dM}{dt}$. In Appendix One we show that this also implies that Condition One is sufficient for $\frac{dF^*}{dt} < 0$ in the exogenous productivity case. Equation (24) implies that $\frac{dF^*}{ds} < 0$ if $\frac{dM}{dt} < 0$. Once again in Appendix One shows that Condition One is sufficient for this in the exogenous productivity case. In summary if condition one holds $\frac{dF^*}{ds} < 0$, $\frac{dG^*}{ds} < 0$, $\frac{dF^*}{dt} < 0$ and $\frac{dG^*}{dt} < 0$ in the exogenous productivity case.

Next we take the case where λ can increase with the mass of firms. The derivatives of (9) in this case are:

$$\begin{aligned}\frac{dF^*}{dt} &= \frac{\partial F^*}{\partial M}\frac{dM}{dt} = \frac{1}{M}\left\{\frac{1}{2}\sqrt{\frac{s^{\frac{1}{s-1}}d}{lM}}\left[1+\frac{dl}{dM}\frac{M}{l}\right]-\frac{d}{l}\frac{M}{l}\frac{dl}{dM}\right\}\frac{dM}{dt} \\ &= \frac{1}{M}\left\{\frac{1}{2}\sqrt{\frac{s^{\frac{1}{s-1}}d}{lM}}\left[1+e_{lM}\right]-\frac{d}{l}e_{lM}\right\}\frac{dM}{dt}\end{aligned} \tag{25}$$

One should also note from (9):

$$1-F^* = \frac{-d}{l}+\sqrt{\frac{s^{\frac{1}{s-1}}d}{lM}} \tag{26}$$

Using this (27) can be written:

$$\frac{dF^*}{dt} = \frac{\partial F^*}{\partial M}\frac{dM}{dt} = \frac{1}{M}\left\{\sqrt{\frac{s^{\frac{1}{s-1}}d}{lM}}\left(\frac{1-e_{lM}}{2}\right)+\left(1-F^*\right)e_{lM}\right\}\frac{dM}{dt} \tag{27}$$

We note that if λ is inelastic with respect to firm entry, i.e. $\varepsilon_{\lambda M}<1$, this ensures that $\operatorname{sgn}\frac{dF^*}{dt}=\operatorname{sgn}\frac{dM}{dt}$. Since $\varepsilon_{\lambda M}<1$ ensures that Condition Two is satisfied, this means that $\varepsilon_{\lambda M}<1$ and condition one are sufficient for $\frac{dM}{dt}<0$ and by implication $\frac{dF^*}{dt}<0$ and $\frac{dG^*}{dt}<0$ in the exogenous productivity case.

$$\frac{dF^*}{ds}=\frac{\partial F^*}{\partial s}+\frac{\partial F^*}{\partial M}\frac{dM}{ds}=\frac{1}{M}\left\{\left(1-F^*\right)e_{lM}+\sqrt{\frac{s^{\frac{1}{s-1}}d}{lM}}\left[\frac{1-e_{lM}}{2}\right]\right\}\frac{dM}{ds}-\frac{1}{2s(s-1)}\sqrt{\frac{s^{\frac{1}{s-1}}d}{lM}} \tag{28}$$

If we are in the exogenous productivity case $\varepsilon_{\lambda M}<1$ and Condition One ensure that both terms in this expression are negative and $\frac{dF^*}{ds}<0$ and $\frac{dG^*}{ds}<0$.

As we noted earlier one we would not argue that the comparative static results here, in particular the result that a higher tax rate reduces the share of the informal sector is a general result. It is nevertheless informative. If we look at equations (6) and (7) we see that the reason the tax rate cancels out in equation (9) is because the tax bill enters the costs of complying firms and the punishment of defaulting firms linearly. If the tax rate in (6) had an exponent greater than unity for example, t would enter (9) and an increase in the tax rate would directly increase the size of the informal sector offsetting the impact of firm exit in increasing the size of this sector. We could think of the comparative static results as illustrating that for plausible parameter values higher tax and enforcement rates typically cause firm exit which in the long run changes the shape of the distribution in a way that increases the share of the informal sector. If there is not a direct affect where the higher tax rate reduces the

share of the informal sector, the impact of firm exit can dominate and the share of the informal sector will increase.[15]

Section II: Data

An important result of our theoretical model is that large firms will operate in the formal sector and will pay higher wages than smaller firms, which are predicted to conduct business in the informal sector, even when there is no heterogeneity amongst either workers or firms ex ante. To investigate whether there is empirical support for these predictions we use the example of South Africa. Our data source is the South African Labour Force Survey (SALFSS). The SALFSS is a twice-yearly rotating panel household survey conducted since September 2000, specifically designed to measure the dynamics of employment and unemployment in the country. For our analysis we use the waves September 2001, March 2002, September 2002, March 2003, and September 2003.[16]

In terms of classifying informal sector activity, the SALFSS explicitly asks individuals that are employed whether their main activity is in the informal sector. More precisely, each employed individual is asked whether 'the organisation/business/enterprise/branch where he/she works is in the formal sector or in the informal sector (including domestic work)'.[17] Additionally, there are a number of other questions regarding fringe benefits of a job that allow us to further verify the individual's informal sector status. These include questions regarding whether the firm is registered, provides medical aid, deducts unemployment insurance contributions, and is registered for VAT. If an individual answers in the affirmative to

[15] In the case of an increase in punishment/enforcement parameter s both the direct and indirect affects go in the same direction.
[16] We restrict our analysis to these waves because they allow us to link households over time.
[17] According to the questionnaire, 'Formal sector employment is where the employer (institution, business or private individual) is registered to perform the activity. Informal sector employment is where the employer is not registered'.

any of these questions, we change his/her sector status to being of the formal sector even if they classify themselves as working in the informal sector.

An important feature of our model described above is that of firm size. In the SALFSS employed individuals provide explicit information on the size of their employer as it falls within six categories: 1 employee, 2–4 employees, 5–9 employees, 10–19 employees, 20-49 employees, 50 or more employees. We create a set of zero-one dummy variables that captures these differences in employer size.

Since we are specifically interested in the pay differential associated with working in the informal sector, an important piece of information required from our data is that concerning remuneration. For those person in paid employment, the SALFSS explicitly asks the remuneration in their main activity. More precisely, the SALFSS provides a person's weekly, monthly, or annual income and hours worked in the previous week in their main job, and we use this information to calculate hourly wage rates.[18] We converted the hourly wage rate data into real wages (September 2001 values) by using the South African consumer price deflator.

An important assumption of our model is that individuals working in the informal sector are not subject to taxation. Ideally we would like to take account of this, however, it is difficult from simple labour force data, where there is no information on non-labour income and where we cannot easily link immediate family members within a household, to accurately estimate the amount on labour income that is likely to be deducted in terms of taxes for most labour market groups. In order to be able to calculate reasonably accurate net (after taxes) income from employment for those working in the formal sector, we thus limit our sample to single men for which

[18] For a small subset of individuals, earnings were only recorded as belonging to predefined amount categories. We excluded these from our analysis.

we can relatively easily infer their income tax liabilities for a given annual income.[19] More precisely, we calculated gross monthly labour income and then used the tax tables relevant for that period as published by the South African Revenue Service to calculate net monthly income for those working in the formal sector and assumed that informal sector workers do not pay taxes on their earnings from employment.[20]

Apart from an explicit definition of the formality of an individual's employer and a precise measure of their remuneration, the SALFSS can also be regarded as relatively rich in other information potentially relevant to an individual's labour market status. We thus compiled information on those factors that are likely to be important for determining a person's pay, as well as whether he/she works in the informal sector. The ones used in the current analysis are grouped for convenience sake into those related to human capital (age, gender, race, marital status, education level, occupation) and job characteristics like job training, region, tenure, and industry (eleven dummies). We provide a comprehensive list of these and their definitions in Table 1.

An important aspect of the data is its rotating panel nature. In this regard, it is easy to link households across waves when they are re-surveyed since they are given a unique household identifier. In contrast, although individuals are likely also to be resurveyed across waves if they remain within the same household, there is no straightforward way to link these across waves. Thus, by pooling all data across waves, we would be using multiple observations across at least some individuals in our analysis without being able to control for this. We thus instead, in order to ensure that this is not the case, only used information taken from one wave per household, arbitrarily chosen as the latest date at which the household was surveyed. Finally, we

[19] One should note that by focusing only on single males allows us also to abstract from the often more complex labour force participation decision that is generally associated with females or married males.
[20] Further details are available from the authors.

reduced our sample to non-self employed males, between the ages of 15 and 70, working in sectors other than the public sector. While comparing self-employed informal sector to their formal sector counterparts may be of interest in its own right, one could argue that the decision of whether to register one's own enterprise is likely to be less constrained or at least determined by different criteria than attempting to get a formal sector job, and thus would require a separate analysis which is beyond the scope of the current paper.

Overall our selection criteria left us with a sample of 7,249 single males of which 1,427 work in the informal sector. We provide some simple summary statistics of these in Table 2. As can be seen, formal sector workers earn substantially more than their informal sector counterparts in terms of gross log wages, namely about 76 per cent. When one allows for the income tax deductions from the earned income for those working in the formal sector, this discrepancy is reduced (to about 54 per cent) but nevertheless remains. We also provide the distribution of the formal and informal sector workers by the given employer sizes in the same table. Accordingly, only about 33 per cent in the formal sector work for firms with less than 10 employees. In contrast, in the informal sector the equivalent figure is about 88 per cent. We also calculated the ratio of the formal relative to the informal log wage rate within firm size categories in Table 3. Here it can be seen that in terms of gross wages the relative log wage rate differences are largest in the very small and the very large employer size categories, while formal sector workers earn between 20 and 35 per cent more in the employer size categories that lie between these two. However, once one allows for tax payments for formal sector workers, the discrepancy is reduced in the largest and the smallest categories, while it virtually disappears for the intermediate ones, especially for those working with employers of size 2-10 workers.

III. Econometric Analysis

Our simple summary statistics suggested that it is important to take account of tax payments by those working in the formal sector when calculating the formal sector wage premium as is assumed in our model. Moreover, comparing wages across the formal and informal sector within categories suggested that at least some of the difference in total mean wages may be due to the different distributions of employer size across the two sectors. This would be supportive of our theoretical result that the formal wage premium may just be due to differences in firm sizes in these two sectors. In order to obtain support for these assertions more formally we now proceed to test them econometrically.

In terms of measuring the wage premium associated with the informal sector one may be tempted to simply run OLS on a standard Mincerian wage equation where one regresses logged wages on an indicator of formal sector employment while controlling for other relevant and available (as from the data) determinants of earnings. However, as recently shown by Pratap and Quintin (2005), not properly taking account of the selection bias in estimating such a parametric regression could bias the results. More specifically, the authors implement a semi-parametric propensity score matching estimator that allows one to explicitly deal with the problem of common support common in standard OLS, where one may be comparing very dissimilar workers. As a matter of fact under OLS Pratap and Quintin (2005) find evidence of a gross wage informal sector premium using Argentinian data, but no such earnings differential is detectable under the semi-parametric propensity score matching estimator. We thus similar follow Pratap and Quintin (2005) and resort to this semi-parametric approach in investigating the formal sector wage premium.

Using a similar notation to Pratap and Quintin (2005) we define the average formal sector premium as what is in the matching literature known as the Average Treatment Effect on the Treated (*ATT*), where treatment refers to employment in the formal sector F:

$$ATT = E\,(wage^F \mid X, sector = F) - E\,(wage^I \mid X, sector = F) \qquad (29)$$

where X are vector of observed individual and job related characteristics and workers i may be employed in the formal sector, $i \hat{I} F$, or in the informal sector, $i \hat{I} I$. If one assumes that the conditional independence assumption holds:

$$wage^F, wage^I \wedge sector \mid X \qquad (30)$$

i.e., that selection only occurs in terms of the observed characteristics, then (29) can be estimated by[21]:

$$ATT = E\,(wage^F \mid X, sector = F) - E\,(wage^I \mid X, sector = I) \qquad (31)$$

Rosenbaum and Rubin (1983, 194) have shown that if the conditional independence assumption holds then conditioning on propensity scores, defined as $P(sector = F \mid X_i)$, is the same as conditioning on the covariates themselves. One can then use these propensity scores to create a sample of `matched' similar individuals, where matching is done via a chosen matching algorithm. In our case we use the caliper method, using a caliper d of size 0.001, although it must be noted that we obtained similar results also using nearest neighbor and kernel matching methods.[22] More specifically, each formal sector worker is matched with a set of informal sector workers whose propensity scores lie within 0.001 of the formal worker in question.

Assuming reasonable matches the *ATT* is then just:

[21] See Rosenbaum and Rubin (1983).
[22] Details are available from the authors upon request.

$$ATT = \frac{1}{N^M}\left(\sum_{i \in F^M}\left(w_i^F - \sum_{i \in I^M}\left(n_{ij} w_i^I\right)\right)\right) \tag{32}$$

where F^M and I^M are the sets of matched formal and informal sector employees, resepectively that could be matched, N^M is the total number of these, and for all *(i, j)* *Î F ´ I*:

$$n_{ij} = \left\{ \begin{array}{ll} 0 & if \quad \left|p_i - p_j\right| > \partial \\ \dfrac{\dfrac{1}{\left|p_i - p_j\right|}}{\sum_{\{i,j:\left|p_i - p_j\right| \le \partial\}} \dfrac{1}{\left|p_i - p_j\right|}} & otherwise \end{array} \right\} \tag{33}$$

In order to generate the propensity score to match formal sector workers we estimate a probit model of formal sector employment conditional on all characteristics as listed in Table 2, alternatively with and without the firm size dummies. Importantly for (33) to be an unbiased estimator of the formal sector wage premium it must be emphasized, however, that the conditional independence assumption must hold and, thus, that one can argue that the set of covariates X that we use to generate the propensity scores captures all factors that determine both selection into formal sector employment and earnings. While it is not possible for us to test this, given our rich set of characteristics we feel reasonably confident that we are indeed likely to be satisfying the conditional independence assumption.

Matching on our set of covariates according to the algorithm above reduced our sample in the case with the firm size dummies to 5,563 and for the one without to 5,587 single men. To assess our success in matching, we, as suggested by Rosenbaum and Rubin (1985), calculated and compared the standardized bias (SB) of the propensity scores for our overall and matched sample using:

$$SB = 100 * \frac{\left|\overline{p_F} - \overline{p_I}\right|}{\sqrt{0.5 * \left(V\left(p_F\right) + V\left(p_I\right)\right)}} \tag{34}$$

where $p_{F,I}$ is the average propensity score and $V\left(p_{F,I}\right)$ its variance for the two sectors. Using this we found that the percentage bias reduction was considerable from matching, around 50 per cent when either including or excluding the firm size dummies. We also, as suggested by Sianesi (2004), compared the pseudo R-squared of our matching equation with the pseudo R-squared from re-estimating this on our matched sample. This was found that to be reduced from 0.41 to 0.14 when we did not include firm size dummies, and from 0.53 to 0.25 when these were included. Thus the matching procedure was able to create a sample for which in terms of our explanatory variables much the decision on participation in the formal sector remains random. In order to see if the matching can be substantially improved with a more restrictive calliper, we also experimented with δ = 0.0001. While this further reduced the sample by about 16 per cent, there was no noticeable reduction in the bias or in lower pseudo r-squared values.

Using our matched sample we then proceeded to calculate the ATT as in (31) first for the gross hourly wage rate without using firm size dummies in the matching procedure, the results of which are given in the first row of Table 4. Accordingly, the earnings premium associated with working in the informal sector is 50.2 per cent and statistically significant. Using net rather than gross wages, as shown in the second row, reduces this premium substantially to 35 per cent, but it still remains statistically significant. Matching with the set of our covariates including the firm size dummies in the subsequent row, the *ATT* on gross wages reduces by 7.7 percentage points, but again lies within standard significance levels. It is only once we assume that informal sector workers do not pay taxes on their wage earnings *and* use firm size dummies in

our matching procedure that the wage premium becomes statistically insignificant. Thus our results suggest, in congruence with our theoretical framework, that in terms of net (of tax) wages, differences in the distribution across employer sizes for informal and formal sector workers and the effect of this firm size wage effect can account for any observed formal sector wage premium.

As a further robustness check we also redid our matching within firm size categories and then calculated out the net wage premium associated with working in the formal sector in the final six rows of Table 4. One should note that this meant matching on small samples, particularly for the very small and the very large categories where there were not many formal and informal sector workers, respectively. Our results show that even within firm size categories there is no significant (net) wage premium. Thus, once one reduces our sample to more homogenous sub-samples in terms of the size of employer there is also no earnings premium for working in the formal sector.

IV. Concluding Remarks

Theory tells us that, while firm size should not affect wages in a competitive labour market, there will be a firm size premium when there are search frictions. In this paper we applied an equilibrium search model to provide a plausible underlying rationale for the duality that many economists have observed in developing countries between small informal low wage firms and large higher wage formal sector firms. Using the South African Labour Force Survey we find empirical evidence supporting the hypothesis implied by our model that firm is a key variable in determining the formal sector wage premium. Our model also shows that because of the impact of firm exit on the shape of the distribution a higher tax rate can reduce the fraction of

non-compliant workers in long run equilibrium. Less surprisingly, an increase in enforcement or punishment of defaulters is found to reduce the size of the informal sector for a wide range of parameter values.

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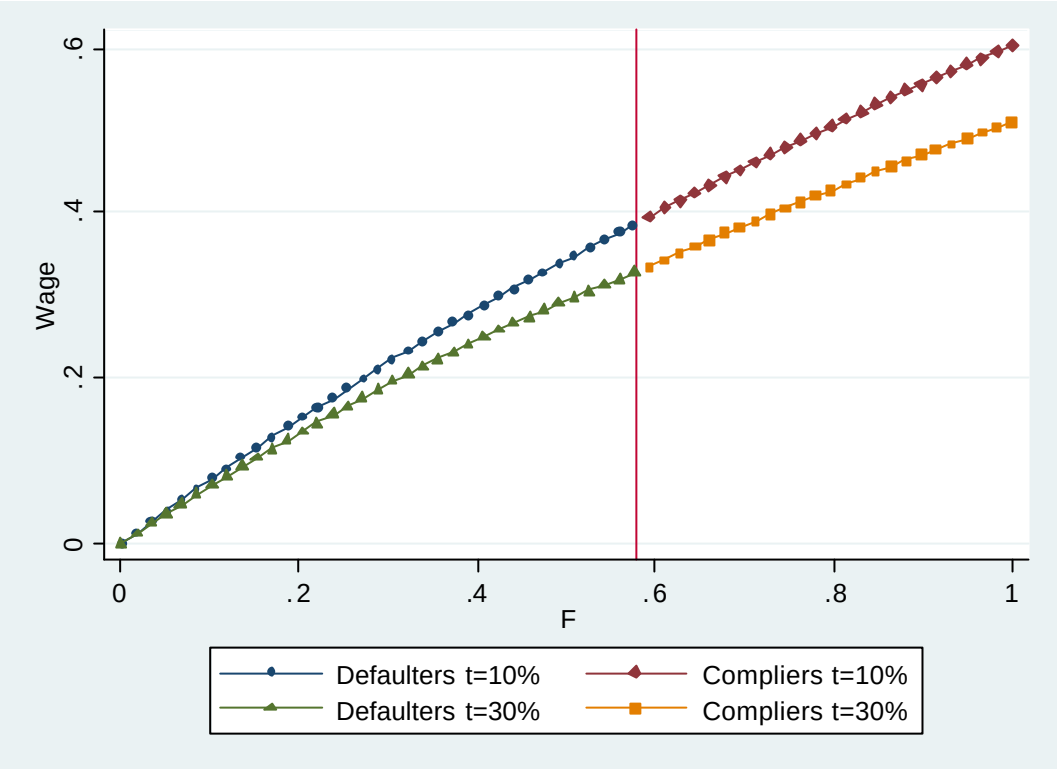
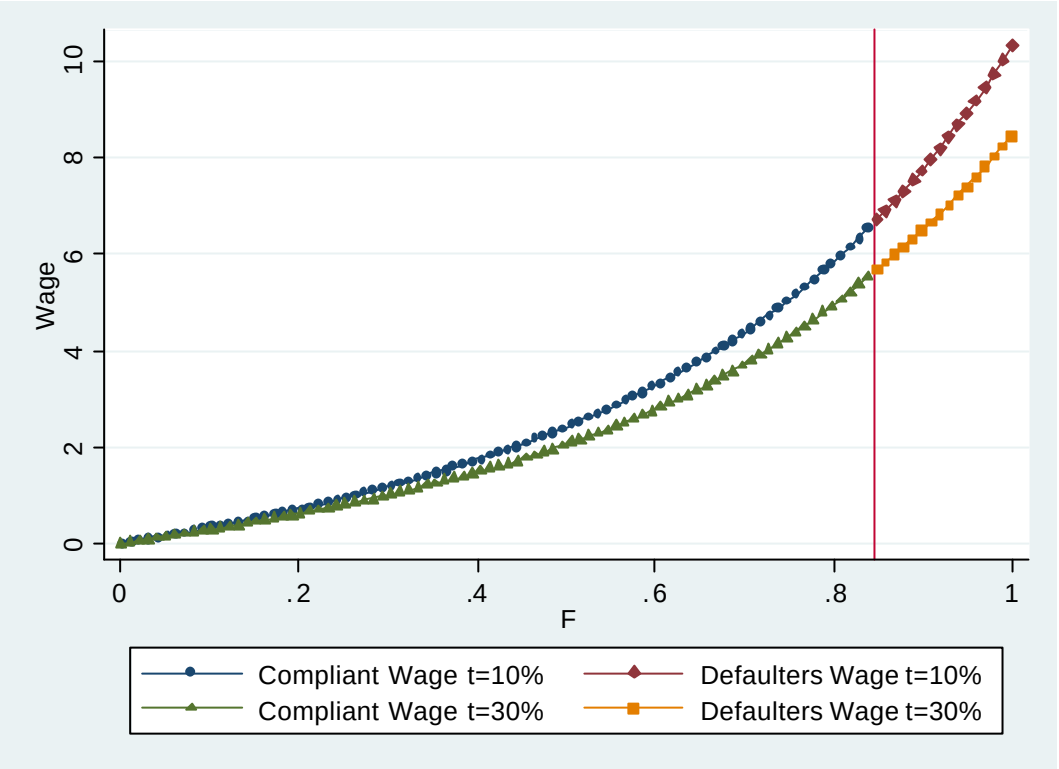
Figure 1: Defaulters and compliers inverse wage offer distributions for tax rates of 10% and 30% and exogenous productivity.



Notes: For both graphs we assume *s=0.2*, *b=0*, *p=1*, and *k=1*, and follow Mortensen (2003) and assume *l=0.287* and *d=0.207*. One should note in particular that the assumption *b=0* simplifies the derivation of *M* and causes the equilibrium mass of firms and cut-off value of F to be constant when t changes in both graphs.

Figure 2: Defaulters and compliers inverse wage offer distributions for tax rates of 10% and 30% and endogenous productivity.



Notes: We make the additional assumption that s=2 for this graph

Table 1: List of Explanatory Variables

Variable name	Definition of the variable
Hourly wage	Real hourly logged wage calculated using a person's income, hours worked in their main job and the South African consumer price deflator
Black White Coloured	Three dummies related to a person's race (the population group that the worker belongs to)
Married	Variable defining the marital status of a person as married
Afrikaans English	Two dummies defining the most often spoken language of the worker at home
No primary (can not read and write), No primary (can read and write), Primary, Secondary, NTC, University	Six dummies associated to a person's education level (the highest level of education completed)
Age	A worker's age (restricted to the interval 15-70)
Job training	The possibility for the worker to be trained in skills that can be used for work
Occupation	Ten dummies for the occupation variables
Urban area	Dummy for whether living in an urban area
Tenure	The period (in years) during which the person was working with the same employer he/she mentioned
Tools	Dummy for whether the person owns the tools and/or the equipment that he/she uses at work
Supervision	Dummy variable for whether the work is supervised
Part-time job	Classifying the job as a full-time job or part-time job (par-time work dummy)
1 worker, 2-4 workers, 5-9 workers, 10-19 workers, 20-49 workers and = 50 workers	Six dummies related to the firm size
Industry	Eleven dummies for the industry variables (eleventh industry dummy 'Exterior organizations and foreign government' is omitted)

Table 2: General Summary Statistics

		Formal	Informal
log(Gross Wage)	*Mean*	1.39	0.79
log(Net Wage)	*Mean*	1.22	0.79
1 employee	*% of total*	0.03	0.49
2-4 employees	*% of total*	0.11	0.38
5-9 employees	*% of total*	0.19	0.11
10-19 employees	*% of total*	0.18	0.06
20-49 employees	*% of total*	0.19	0.03
50+ employees	*% of total*	0.25	0.01

Table 3: Ratio of the Formal Relative to the Informal log Wage Rate by Employer Size

Firm Size	log(Gross Wage) Ratio	log(Net Wage) Ratio
1 employee	1.66	1.38
2-4 employees	1.28	1.07
5-9 employees	1.21	1.05
10-19 employees	1.34	1.17
20-49 employees	1.34	1.17
50+ employees	1.92	1.71

Table 4: Estimate of ATT of the Formal Sector Wage Premium

Sample	Wage	Firm Size DVs Included	ATT	Standard Error	Matched Sample
Total	*Gross*	*No*	0.502**	0.057	5587
Total	*Net*	*No*	0.350**	0.055	5587
Total	*Gross*	*Yes*	0.423*	0.186	5563
Total	*Net*	*Yes*	0.241	0.157	5563
1 employees	*Net*	---	0.010	0.228	126
2-4 employees	*Net*	---	0.010	0.103	451
5-9 employees	*Net*	---	-0.006	0.157	367
10-19 employees	*Net*	---	-0.079	0.156	449
20-49 employees	*Net*	---	0.020	0.244	366
50+ employees	*Net*	---	0.299	0.515	51

Notes: (1) ** and * stand for one and five per cent significance levels, respectively. (2) Standard errors generated via boostrapping using 500 replications. (3) Matching done separately for individual firm size categories.

Appendix One: The Impact of a Change in *t* or *s* on the Mass of Firms

Setting (13) equal to zero and totally differentiating with respect to t and M we get the following expression:

$$\begin{aligned}&\{\frac{\partial k}{\partial M}\\&+[\frac{(p-b)n(b)}{d}-s\ \frac{bstn^s(b)}{d}]\frac{1}{M}\\&+[\frac{(p-b)n(b)}{d}-s\ \frac{bstn^s(b)}{d}][\frac{(l-d)\}}{d+l}]\frac{\partial l}{\partial M}\frac{1}{l}\}dM\\&+\frac{bsn^s(b)}{d}dt=0\end{aligned} \qquad \text{(A.1.1)}$$

The first line constitutes the change in fixed entry costs from a change in the mass of firms, the second term is the direct impact of a change in the mass of firms, the third line is the derivative from a change in offer arrival rates resulting from a change in firm entry, and the fourth line provides the derivative with respect to a change in the tax rate *t*. One should note that if one totally differentiates (13) with respect to the punishment/enforcement rate *s* and *M* one would get the same expression as (16) except that the final term would be: $\frac{btn^s(b)}{d}ds$. Also, we assume that $\frac{\partial k}{\partial M}>0$ and $\frac{\partial l}{\partial M}>0$ and define $\frac{\partial l}{\partial M}\frac{M}{l}=e_{lM}$ as the elasticity of the arrival rate with respect to firm. We can multiply (A.1.1) by *M* and rewrite it as:

$$\begin{aligned}&\{\frac{\partial k}{\partial M}M\\&+[\frac{(p-b)n(b)}{d}-s\ \frac{bstn^s(b)}{d}][\frac{l(1+e_{lM})+d(1-e_{lM})\}}{d+l}]\}dM\\&+\frac{bsn^s(b)}{d}dt=0\end{aligned} \qquad \text{(A.1.2)}$$

First we will take the left hand side term in square brackets from the second line of (A.1.2):

$$\frac{(p-b)n(b)}{d} - s\,\frac{bstn^{s}(b)}{d} \qquad \text{(A.1.3)}$$

We note from (9) that in any equilibrium where there are some defaulting firms $s < \frac{1}{n^{s-1}(b)}$. Substituting the right hand side in for s in (A.1.3) we see that a sufficient condition for this expression to be positive is:

Condition One: $p > b(1+st)$

Next one can say that if Condition One holds then $\frac{dM}{dt} < 0$ if condition two holds, where:

Condition Two: $\frac{l}{d} > \frac{e_{lM} - 1}{1 + e_{lM}}$

While Condition Two may not hold, for reasonable parameter values the indication is that it will hold unless $e_{?M}$ is very large. For example if ?>d Condition Two certainly holds, or, taking the values ?=0.207 and d=0.287 used by Mortensen (2003) in his simulations, Condition Two will hold as long as $e_{?M}$<6.17. One should also note that this is a sufficient condition, so there is a range of parameter values where Conditions One or Two fail but $\frac{dM}{dt} < 0$ continues too hold. We also remark that in the simpler case where ? is not dependent on the mass of firms Condition Two always holds so that Condition One is sufficient for $\frac{dM}{dt} < 0$.